THE NON-LINEAR REGRESSION – THE LEVENBERG-MARQUARDT ALGORITHM FOR ASSUMPTION THE ENERGY LOSSES OF HYDRAULIC TRANSPORT IN A CASE OF THE MINE "TREPCA"

by

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The main problem of hydraulic transport is the resistance generated during the mixture transport through the pipe-line. Testing the flow characteristics of mixtures, shown in this paper, are based on the principles of determining the unit energy losses by a mathematical calculation using the non-linear regression – the Levenberg-Marquardt algorithm. Such obtained results allow determining a transport rate in the horizontal pipe-line, depending on the mixture bulk density and pipe-line diameter. The flotation tailings is mainly used as a filling material in the mine "Trepca" - Stari Trg. According to the grain size distribution, it is a fine-grained material of a size of 0.074 mm to 1.2 mm. It is a multicomponent material containing pyrite, pyrrhotine and other heavy metals, and therefore, has a high bulk mass. The average rate of the hydromixture, in which the energy losses reach the minimum value, depends on the pipe-line diameter and kinetic bulk density of the mixture. For the test interval of change in the pipe-line diameter, shown in this paper (0.168 mm, 0.176 mm, 0.193 mm, and 0.225 mm), and kinetic bulk density of the hydraulic mixture (1-1.6 kg/m^3), this rate ranges from 3-5.5 m/s. The increase of the energy losses in the hydraulic mixture transport increases proportionality with the increase of its kinetic bulk density. The results, presented in this paper, show that the required bulk density of 1.6 kg/m^3 should be accepted as a limit from a point of view of the hydraulic transport cost-efficiency.

Key words: *hydro-filling, flotation tailings, energy losses, transport, resistance, pipe-line*

Introduction

Approximately 14 billion tonnes of tailings were produced globally by the mining industry in 2010. The need for a comprehensive framework for mine tailings management (including dewatering) that promotes sustainable development is therefore, becoming increasingly recognised by the mining industry [1]. The expenses for hydraulic transportation of solid materials are a complicated function of mechanical characteristics of solid phase and hydraulic fluid [2]. Additional assessment of pumping and processing technology systems

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will provide a more complete estimation of energy and water usage in a particular tailings disposal strategy.

Environmental and economic perspectives should also be considered in the assessment [3]. This framework will be able to guide the mining sector to choose its mine tailings management strategy based on sustainable development concepts [1]. Examining the trade-off between water and energy use in tailings management is a good start in improving both the efficiency and sustainability performance of mine tailings management [3]. When designing hydrotransport systems, the issues of solids concentration in a hydraulic fluid-flow must be solved, which will result in the lowest possible energy consumption [2].

According to history of investigation, experimental techniques have been developed for the measurement of two important parameters-particle concentration and liquid velocity, in order to obtain a better understanding of the transport mechanism [4]. The flow of hydromixture, made of flotation tailings and water, in the horizontal pipe-lines of a certain diameter, depends on a number of parameters, out of which the most important are the following: volume concentration of solid material in mixture, average transport speed, physical-mechanical characteristics of the material and energy losses during transport.

For the mixture of water and tested flotation tailings, those values of the average transport rate, volume concentration of solid material and pipe-line diameter should be selected, which would allow an hydraulic transport capacity to be achieved with the minimum energy consumption during transport without causing material sedimentation [5, 6]. If the flow rate is sufficiently high, turbulence is effective in keeping all the solids suspended (fully suspended flow); otherwise the particles accumulate at the pipe bottom and form a packed bed, either sliding (flow with a moving bed) or not (flow with a stationary bed) [7].

Group of authors give an example for hydraulic transport of very large solid particles (above 5 mm) in an horizontal pipe where mixtures of size and/or specific mass are studied, along with characterization of differential pressure measurements and visualizations [8]. A recent study reported an experimental investigation of the transport of fluids composed of water and small size polypropylene particles, in order to study the transport of floating particles, also to determine the conditions that minimize the energy consumed [9]. The flow of solid-liquid mixtures is considered as a very complex process [7, 9].

In order to find the values for economical and stable hydraulic transport of flotation tailings from the ore processing plant (tailing dump), it is necessary to carry out the tests of physical-chemical properties of the flotation tailings of the lead and zinc ore. These are primarily the grain size distribution and chemical composition as well as the bulk density. It is also necessary to determine the critical transport rates for various values of the volume concentrations and pipe-line diameter, measurement the energy losses in the pipe-line of different diameter, for different values of the volume concentration and mean rate of the hydro mixture transport.

The analysis of parameters, changed in the pipe-line during the hydraulic transport, as well as mathematical interpretations, represent the basis for selection the parameters of the hydraulic transport of water and flotation tailings mixture in the installation of hydraulic filling in the mine. The dynamic development of advanced numerical techniques observed in recent years together with constantly increasing computational efficiency of computers have a great influence on the development of various methods of mathematical modelling of measurement systems [10, 11]. Many researchers have tried to create a mathematical model in order to predict the variable parameters in slurry transport [9]. Recent workings show that numerous authors are showing various models applied for certain calculation [12-15].

For the hydraulic transport process, without the field measurements, the mathematical model of the Levenberg-Marquardt algorithm can be used [16, 17]. The aim of such modelling is to synthesize models that map precisely the dynamic properties of real systems [10]. According to Tomczyk, one signal can be used as a value which maximizes the assumed objective function as an alternative for analyzing the full set of all imaginable input dynamic signals [18, 19].

In this paper the authors presented basic information about material which is used to make hydro-mass for filling, the description of mathematical model used for determination the parameters of the hydraulic transport, as well as comments of results and conclusion. The aim of this study was to show that by using of the mathematical model of the Levenberg-Marquardt algorithm the parameters of the hydrotransport can be determined without measuring the field.

Experimental part

Material characteristics for hydro transport

The material, represented a solid phase for formation the hydro-mixture, was obtained as a waste material during the flotation process of the lead and zinc ore in the Mining-Metallurgical-Chemical Combine "Trepca" in Zvecan. Namely, this is a mean sample of a fine fraction of material grain that can be successfully applied in a form of hydro-mixture for filling the excavations in the mines. [20, 21]

Chemical composition of the material for hydro-mixture is shown in tab.1.

Table 1. Chemical composition of the material for hydro-mixture

Component	Pb	Zn	Fe	S	Mn	CaO	SiO_2	Al_2O_3	MnO
Content [%]	0.22	0.27	20.70	10.75	3.67	42.80	10.58	7.19	3.03

A flotation tailings sample contains 14% of pyrites and 4.5% of pyrrhotine that is within the allowed application limits in the hydro-filling process. The specific mass of the flotation tailings, depending on the chemical composition, was $3200 \leq \rho_s \leq 3390$ kg/m^3. The grain size distribution analysis was carried out on tailings from the flotation concentration process, and the obtained results are shown in tab. 2, and graphically in fig. 1.

Table 2. Grain size distribution of flotation tailings

Size class [mm]	M [%]*	M [%]↓**	M [%]↑***
+1.175	0.20	0.20	100.00
–1.175 + 0.833	0.80	1.00	99.80
–0.833 + 0.587	4.00	5.00	99.00
–0.587 + 0.417	9.10	14.10	95.00
–0.417+0.295	11.90	26.00	85.90
–0.295+0.208	14.50	40.50	74.00
–0.208+0.147	15.70	56.20	59.50
–0.147+0.104	13.20	69.40	43.80
–0.104+0.074	9.80	79.20	30.60
–0.074+0.00	20.80	100.00	20.80

*M, % – weight percent of size fractions in sample, ** M, %↓ - cumulative oversize, *** M, %↑ - cumulative undersize

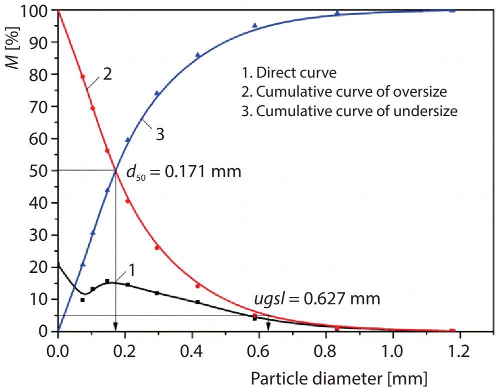

Figure 1. Curves of grain size distribution of flotation tailings

The mean grain diameter, d_{50} = 0.171 mm, and upper grain limit size class, *ugsl* = 0.627 mm, were read from a diagram of the grain size distribution.

From the aspect of application of tested material it is important to determine these data. Generally, tailings that remains after the flotation concentration process must be adequate grain size in order to use as a sand for hydraulic filling in mines. For requirements of Mining-Metallurgical-Chemical Combine "Trepca" in Zvecan the material that represents a solid phase must be fine-grained fractions of material that represent tailings, that one can be used for generation of hydraulic mixture. Values of the mean grain diameter, d_{50} = 0.171 mm, and upper grain limit size class, *ugsl* = 0.627 mm show that previously mentioned condition is accomplished.

Methods

Model parameters were determined using the non-linear regression (Levenberg-Marquardt algorithm). The non-linear regression is a form of the regression analysis in which it is necessary to determine a function (model) with a set of parameters that best fits the data.

In the regression model in which the function and set of initial parameters are defined, the best values of parameters can be obtained using the software. The greatest role in determining the dependency of variables, which are analyzed using the software (by implementation the appropriate algorithms), is still on the side of a person who has to assume the equation of dependence.

Coefficients in the equation (unknown parameters) are determined using the software R (which uses the Levenberg-Markvart algorithm)[**]. The Levenberg-Markvart (Levenberg-Marquardt - LM) algorithm applies to the advanced and fastest standard algorithm for the non-linear optimization.

This model also has applications in more areas of science, for calculating parameters relevant for monitoring certain processes. [16, 17, 22]. The authors used instructions given in the literature referring to this mathematical model in order to facilitate the analysis of the obtained data. [23, 24]. That mathematical model is also known as the probably method of damped least squares. This algorithm uses an iterative improvement of the parameter values to reduce the sum of squared differences between the experimental and

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[**] https://cran.r-project.org/web/packages/minpack.lm/minpack.lm.pdf

values estimated by a model. The LM algorithm uses the steepest descent method***, when the parameter values are far from optimum, while the Taylor series method [16] is used in the case when the parameter values approach their optimal values. Therefore, the process of calculating the parameters is iterative. To determine the parameters of this model (a, b), the programming language R was used. The R is a program language and integrated programming environment for data management, but also for their analysis (numerical, statistical) and graphical representation [24, 25]. Besides providing a wide variety of statistical tests, the R allows users to add the new functionalities defining the new functions. The R is expanded using concept of packages as extension modules under open-source, GNU General Public License. Its strong relationship to the academic community actively contributes to the R to follow the trends in development the new packages. The number of developed packages exponentially increases, and is currently more than 1000. All packages are available on the CRAN (The Comprehensive R Archive Network) web page, and are located in the thematically grouped packages [26].

Results and discussion

The energy losses during hydrotransport, using the mathematical model shown in this paper, were obtained in changing the following parameters:

$$v = 1\text{-}10, \quad \gamma_m = 1.0\text{-}1.6, \quad C_V = 0\text{-}0.22$$

where v [ms^{-1}] – the flow rate of hydraulic mixture, γ_m [kgm^{-3}] – the bulk density of mixture, C_V [m^3 transported material / m^3 volume of the pipe-line] – the volume concentration of solid material grains in hydraulic mixture.

The non-linear regression application (Levenberg-Marquardt algorithms) defines the parameters of the model without measuring in the field for diameters: 168 mm, 176 mm, 193 mm, and 225 mm.

The model equation is represented by the formula:

$$i_m = \left(aV_m^2 + b\frac{\gamma_m - \gamma_w}{V_m} \right) 10^{-4} \tag{1}$$

where i_m [mVSm^{-1}] – the energy loss during transport of hydromixture (unit loss-energy consumption overcome the resistance on the route), a and b – the model parameters, γ_m [kgm^{-3}] – the bulk density of mixture, γ_w [kgm^{-3}] – the bulk density of water, V_m [m^3] – hydromixture volume.

Using the eq. (1), the energy losses were calculated for the different transport time of hydromixture in the horizontal pipe-lines, 2-10 m/s and different diameters D = 168 mm, 176 mm, 193 mm, and 225 mm. Based on the obtained data, tabs. 3-6, the energy loss diagrams were drawn and shown in figs. 2-5. The formed curves give a view of energy losses in the pipe-lines with exactly defined nominal pipe diameter from the subcritical to the supercritical transport rates.

The model parameters are determined by the non-linear regression: a = 38.35 and b = 9634.79 so the model equation is:

$$i_m = \left(38.35V_m^2 + 9634.79\frac{\gamma_m - \gamma_w}{V_m} \right) 10^{-4} \tag{2}$$

[***] The steepest descent method is a gradient method in which a step size is selected so that a maximum decrease in the value of target function is achieved in each iteration (step) of the method

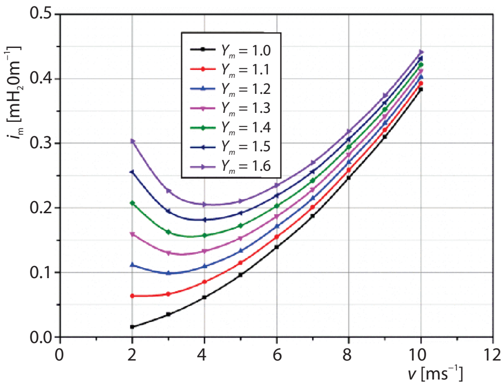

Figure 2. Energy loss during the transport of hydromixture through the pipe-line *D* = 168 mm, depending on the medium transport rate and bulk density

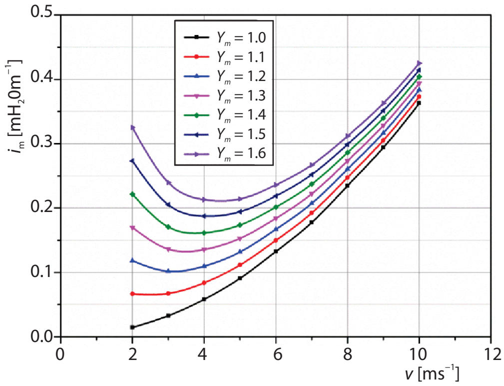

Figure 3. Energy loss during the transport of hydromixture through the pipe-line *D* = 176 mm, depending on the medium transport rate and bulk density

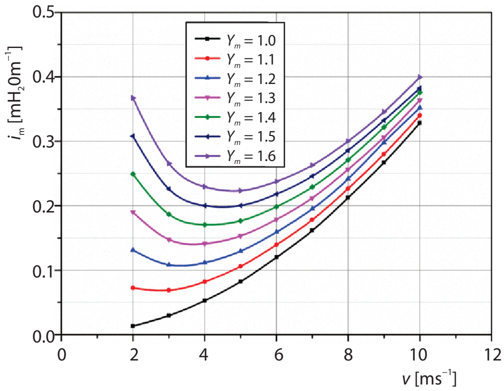

Figure 4. Energy loss during the transport of hydromixture through the pipe-line *D* = 193 mm, depending on the medium transport rate and bulk density

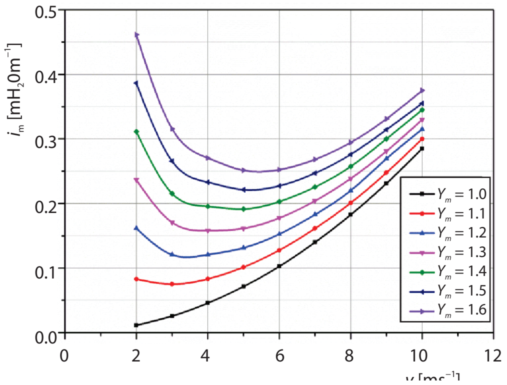

Figure 5. Energy loss during the transport of hydromixture through the pipe-line *D* = 225 mm, depending on the medium transport rate and bulk density

Table 3. Energy losses in the flow of hydromixture, expressed in meters of a water column per meter length of the pipe-line for a diameter of 168 mm in a horizontal installation

D = 168	V_m								
γ_m	2	3	4	5	6	7	8	9	10
1	0.0153	0.0345	0.0612	0.0958	0.1390	0.1872	0.2465	0.3099	0.3835
1.1	0.0635	0.0666	0.0852	0.1150	0.1550	0.2010	0.2585	0.3206	0.3931
1.2	0.1114	0.0986	0.1092	0.1332	0.1711	0.2148	0.2705	0.3312	0.4028
1.3	0.1595	0.1306	0.1332	0.1534	0.1868	0.2286	0.2825	0.3419	0.4124
1.4	0.2075	0.1627	0.1572	0.1726	0.2031	0.2424	0.2944	0.3526	0.4220
1.5	0.2556	0.1947	0.1814	0.1919	0.2191	0.2562	0.3064	0.3632	0.4316
1.6	0.3036	0.2267	0.2052	0.2105	0.2350	0.2700	0.3184	0.3739	0.4412

Table 4. Energy losses in the flow of hydromixture, expressed in meters of a water column per meter length of the pipe-line for a diameter of 176 mm in a horizontal installation

$D = 176$	V_m								
γ_m	2	3	4	5	6	7	8	9	10
1	0.0145	0.0327	0.0580	0.0907	0.1325	0.1775	0.2345	0.2939	0.3628
1.1	0.0665	0.0672	0.0838	0.1114	0.1498	0.1924	0.2474	0.3053	0.3731
1.2	0.1180	0.1017	0.1097	0.1317	0.1670	0.2073	0.2603	0.3168	0.3835
1.3	0.1698	0.1362	0.1355	0.1529	0.1841	0.2222	0.2732	0.3283	0.3938
1.4	0.2216	0.1707	0.1614	0.1736	0.2015	0.2371	0.2861	0.3398	0.4042
1.5	0.2733	0.2052	0.1875	0.1943	0.2188	0.2520	0.2989	0.3513	0.4145
1.6	0.3251	0.2398	0.2131	0.2141	0.2360	0.2669	0.3118	0.3627	0.4249

Table 5. Energy losses in the flow of hydromixture, expressed in meters of a water column per meter length of the pipe-line for a diameter of 176 mm in a horizontal installation

$D = 193$	V_m								
γ_m	2	3	4	5	6	7	8	9	10
1	0.0131	0.0296	0.0527	0.0823	0.1199	0.1614	0.2123	0.2669	0.3285
1.1	0.0723	0.0689	0.0821	0.1059	0.1395	0.1783	0.2267	0.2800	0.3403
1.2	0.1311	0.1082	0.1116	0.1295	0.1592	0.1952	0.2417	0.2981	0.3521
1.3	0.1901	0.1476	0.1410	0.1531	0.1788	0.2121	0.2564	0.3062	0.3639
1.4	0.2491	0.1868	0.1705	0.1767	0.1985	0.2290	0.2711	0.3218	0.3757
1.5	0.3081	0.2261	0.2001	0.2003	0.2181	0.2459	0.2858	0.3324	0.3825
1.6	0.3671	0.2656	0.2294	0.2233	0.2377	0.2628	0.3002	0.3455	0.3993

Table 6. Energy losses in the flow of hydromixture, expressed in meters of a water column per meter length of the pipe-line for a diameter of 225 mm in a horizontal installation

$D = 225$	V_m								
γ_m	2	3	4	5	6	7	8	9	10
1	0.0114	0.0257	0.0457	0.0713	0.1026	0.1399	0.1825	0.2310	0.2850
1.1	0.0829	0.0751	0.0831	0.1013	0.1276	0.1613	0.2009	0.2477	0.3000
1.2	0.1614	0.1206	0.1206	0.1313	0.1525	0.1827	0.2199	0.2693	0.3150
1.3	0.2364	0.1706	0.1580	0.1613	0.1775	0.2041	0.2386	0.2810	0.3300
1.4	0.3114	0.2155	0.1955	0.1913	0.2029	0.2255	0.2573	0.3001	0.3450
1.5	0.3864	0.2654	0.2329	0.2213	0.2273	0.2469	0.2760	0.3143	0.3550
1.6	0.4614	0.3156	0.2704	0.2513	0.2524	0.2683	0.2945	0.3309	0.3750

The model parameters are determined by the non-linear regression: $a = 36.34$ and $b = 10409.35$ so the model equation is:

$$i_m = \left(36.34 V_m^2 + 10409.35 \frac{\gamma_m - \gamma_w}{V_m} \right) 10^{-4} \tag{3}$$

The model parameters are determined by the non-linear regression: $a = 32.94$ and $b = 11847$ so the model equation is:

$$i_m = \left(32.94 V_m^2 + 11847 \frac{\gamma_m - \gamma_w}{V_m} \right) 10^{-4} \tag{4}$$

The model parameters are determined by the non-linear regression: a = 28.59 and b = 14566.2 so the model equation is:

$$i_m = \left(28.59V_m^2 + 14566.2\frac{\gamma_m - \gamma_w}{V_m}\right)10^{-4} \tag{5}$$

In order to explain, as fully as possible, the energy loss diagrams, shown in figs. 2-5, it was necessary to determine a range of the critical flow rates of hydromixture in the pipes on the basis of minimum values of energy losses. Namely, the movement of hydromixture through the horizontal pipe-line without the solid phase precipitation is only possible in the conditions where the mean flow rate is higher than the critical rate, which is usually 6-20% depending on the material.

The minimum energy consumption in the transport with pipes is achieved by movement the hydromixture with critical rate, and therefore, in practice, the rate should be maintained as near as possible to the critical rate, but a sufficient high to avoid precipitation [27].

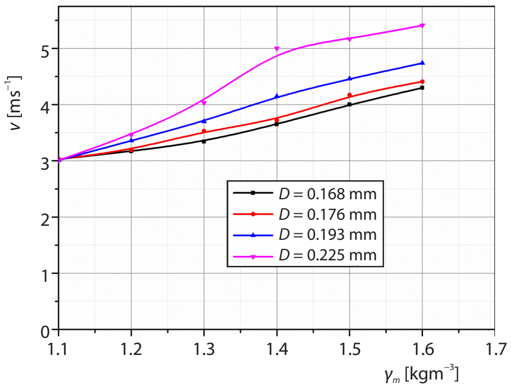

Figure 6. Determining the critical flow rate of hydromixture in the pipes on the basis of the minimum values of energy losses depending of bulk density of mixture

Taking into account that determining the critical rate is extremely important for the hydrotransport with pipes, the critical flow rates are determined in this work for all tested pipe diameters. Data were used for the rate of realized lowest energy loss, read from the graphics in figs. 2-5 for all bulk densities of the mixture. The resulting diagram is shown in fig. 6.

Diagrams presented in fig. 6 show that the range of critical rates for tested pipe--lines in this work is 3 m/s, which is the lowest value for all pipe diameters up to 5.3 m/s for D = 0.225 mm.

On the basis of the obtained energy loss diagrams, figs. 2-5, it can be seen that each curve (D = const, γ_m = const) has the form of parabola with a left steep branch. The points on the left branch of curve corresponding to the subcritical transport rates get less and less values of ordinances depending on the growth of abscissae (rates). Transport of hydromixture in these conditions takes place by deposition the grains on the bottom and their rejection on the pipe-line walls [28, 29]. In the vicinity of the point corresponding to the extreme value of the energy loss function, depending on the average rate, and kinetic specific rate of hydromixture, a relatively mild flow of the curve is observed without a pronounced minimum value. This indicates that there is a large flow rate interval in which, with significant differences in relation the critical rate V_{kr} (minimum values), the material does not precipitate in the pipe-line nor create the conditions for its blockage. The critical rate ranges within the limits from 3-5.5 m/s, fig. 6, for the conditions shown in this paper, which are the diameter of pipe-line 0.168 mm, 0.176 mm, 0.193 mm, and 0.225 mm, and kinetic bulk density of hydraulic mixture 1-1.6 kg/m^3.

Growing branch of the energy loss curve of hydromixture flow, on all diagrams, corresponding to the above supercritical transport rates, at higher rates, receives the values approximate to the curve of water losses (γ_m = 1 kg/m^3). Namely, the energy loss curve of hydromixture is approaching the asymptotic curve of water losses. This is a confirmation of the already known principle that with the increase in the hydromixture flow rate, the losses, caused by the grain transport of contained solid material, are reduced.

The hydromixture energy loss curves are located one above the other at distances proportional to the bulk density of hydriomixture (D = const), and obtain the lower values of ordinates (losses) for the same abscissae (rates), and critical specific masses of hydraulic mixture for larger diameters of the pipe-line.

Conclusions

Based on the results presented in this paper, the following can be concluded:

- In hydraulic transport of the flotation tailings in horizontal pipe-lines, the energy flow losses are greater than the losses in water flow considering the difference in these losses depends on the volume concentration and average flow rate of hydromixture. The curve of energy losses characterizing the flow of analyzed hydromixture resembles in its shape on a parabola whose one branch is moved to the horizontal. The left part of curve corresponds to the subcritical rates and, in that case, the hydromixture is transported with the deposition of grains on the bottom and their periodical removal. Hydraulic transport is not recommended for such rates due to the risk of pipe-line blockage, as well as the increased pipe-line abrasion. The average rate of hydromixture, in which the energy losses reach the minimum value, is a critical rate depending on the pipe-line diameter and kinetic bulk density of the mixture. For the test interval of change in the pipe-line diameter, shown in the work (0.168 mm, 0.176 mm, 0.193 mm, and 0.225 mm), and kinetic bulk density of the hydraulic mixture (1-1.6 kg/m^3), this rate changes within the limits from 3-5.5 m/s. The right part of energy loss curve is characterized by the flow with supercritical rates, and the size of energy losses increases in parallel with the rate increase.
- With the increase in flow rate, the increase in energy losses is getting smaller, so at high rates, the curves of hydromixture energy losses increasingly asymptotically approach the curve of water losses.
- The increase in energy losses in the hydraulic mixture transport increases proportionally to the increase in its kinetic bulk density. Therefore, it is necessary to accept the bulk density of 1.6 kg/m^3 as the limit from the point of view of cost-effectiveness of the hydraulic transport.
- The results presented in this paper, obtained by the mathematical calculation using the non-linear regression – the Levenberg-Marquardt algorithm and made conclusions, are related to the hydromixtures with low-volume concentrations, with a precisely determined spectrum of grain-size, as well as to the hydrotransport with horizontal pipe-lines with the constant diameters and rough walls. Considering that the tests indicated the transport conditions with the lowest energy losses, so they can be also applied in practice in the other materials.
- Namely, determining the hydraulic transport parameters of the fine-grained and coarse-grained hydromixtures with different solid particle granulation can be also applied in practice in designing a hydraulic transport of materials with properties similar to those of the investigated flotation tailings of the lead and zinc ore.
- Generally, the greatest contribution of the research shown in this paper is that the authors have shown that the application of the non-linear regression - the Levenberg-Marquardt algorithm can determine the parameters of hydraulic transport without field measuring, which extremely speed up the planning process.

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