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# A backward/forward sweep power flow method for harmonic polluted radial distribution systems with distributed generation units 

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## Summary

This paper presents a Backward/Forward Sweep (BFS) power flow method in the frequency domain for the analysis of the harmonic distortion in radial distribution systems with distributed generation (DG) units. A detailed procedure for solving the power flow problem at fundamental and harmonic frequencies and the models of distribution system elements in harmonic analysis are presented. In addition, the effects of converter-based and nonconverter-based DG


#### Abstract

LIST OF SYMBOLS AND ABBREVIATIONS: $\left(\underline{I}_{l, i}^{(h)}\right)^{(k)}$, Harmonic current of linear load $i$ for iteration $k$; $\underline{Y}_{i}^{(1)}$, Fundamental admittance of line/ cable $i ;\left(\underline{I}_{d g, i}^{(1)}\right)^{(k)}$, Fundamental current of the DG at bus $i$ for iteration $k ;\left(\underline{I}_{l, i}^{(1)}\right)^{(k)}$, Fundamental current of the linear load at bus $i$ for iteration $k$; $\left(\underline{I}_{n l, i}^{(1)}\right)^{(k)}$, Fundamental current of the nonlinear load at bus $i$ for iteration $k ;\left(\underline{J}_{i}^{(1)}\right)^{(k)}$, Fundamental current through branch $i$ for iteration $k$; $\underline{Z}_{i}^{(1)}$, Fundamental impedance of line/cable $i ; \underline{Y}_{s h, i}^{(1)}$, Fundamental shunt admittance of elements incident to bus $i ;\left(U_{i}^{(1)}\right)^{(k)}$, Fundamental voltage amplitude at bus $i$ for iteration $k$; $U_{0}^{(1)}$, Fundamental voltage amplitude of the power supply bus; $\left(\theta_{i}^{(1)}\right)^{(k)}$, Fundamental voltage phase angle at bus $i$ for iteration $k ; \underline{Y}_{i}^{(h)}$, Harmonic admittance of line/cable $i ; \underline{Y}_{d g, i}^{(h)}$, Harmonic admittance of the linear DG at bus $i ; \underline{Y}_{l, i}^{(h)}$, Harmonic admittance of the linear load at bus $i ; I_{n l, i}^{(h)}$, Harmonic current amplitude injected by the nonlinear load at bus $i ;\left(I_{d g, i}^{(h)}\right)^{(k)}$, Harmonic current injected by the nonlinear DG at bus $i$ for iteration $k ;\left(\underline{I}_{n l, i}^{(h)}\right)^{(k)}$, Harmonic current injected by the nonlinear load at bus $i$ for iteration $k ;\left(\underline{J}_{i}^{(h)}\right)^{(k)}$, Harmonic current through branch $i$ for iteration $k ; \underline{Z}_{i}^{(h)}$, Harmonic impedance of line/cable $i ; \underline{Y}_{s h, i}^{(h)}$, Modified harmonic shunt admittance of elements incident to bus $i$; $\theta_{n l, i}^{(h)}$, Phase angle of the harmonic current injected by nonlinear load at bus $i ; \underline{S}_{i}^{(h)}$, Power flow in branch $i ; \underline{S}_{\text {loss }, i}^{(h)}$, Power losses in branch $i$; $\mathbf{Q}_{d g}$, Reactive power injection vector of PV buses; $\underline{\mathbf{Z}}_{P V}$, Sensitivity impedance matrix of PV buses; $P_{d g, i}^{s p}$, Specified active power of the DG at bus $i$; $Q_{d g}^{\min }, Q_{d g}^{\max }$, Specified generator limits; $Q_{d g, i}^{s p}$, Specified reactive power of the DG at bus $i ; \underline{U}_{d g}^{s p}$, Specified voltage at the PV bus; $\underline{\mathbf{U}}_{d g}^{s p}$, Specified voltage vector of PV buses; $X_{d g, i}^{\prime \prime}$, Sub-transient reactance of the DG at bus $i ; \underline{S}_{l o s s}^{(h)}$, Total power losses of the system; $\theta_{n l, i}^{(h-s p e c t r u m)}$, Typical phase angle of the harmonic source current spectrum; $\underline{\mathbf{U}}_{d g}$, Vector of calculated voltages of PV buses; $\left(\underline{U}_{i}^{(1)}\right)^{(k)}$, Voltage at bus $i$ for iteration $k ; \underline{U}_{d g}^{(k)}$, Voltage at the PV bus for iteration $k$; $\left(\underline{U}_{i}^{(h)}\right)^{(k)}$, Harmonic component of the voltage at bus $i$ for iteration $k$; ASD, Adjustable-speed drive; BFS, Backward/Forward Sweep; $C(h)$, Ratio of the $h$-th harmonic current to its fundamental value; CHPF, Coupled harmonic power flow; $C_{i}$, Capacitance of line/cable $i$; DG, Distributed generation; DHPF, Decoupled harmonic power flow; $f$, Frequency; $h$, Harmonic order; $h_{\text {max }}$, Highest harmonic order of interest; HPF, Harmonic power flow; $k$, Iteration number; KCL, Kirchhoff's current law; $k_{\text {max }}$, Maximum number of iterations; KVL, Kirchhoff's voltage law; $L_{i}$, Inductance of line/cable $i$; $n$, Any integer ( $1,2, \ldots$, etc.); $N$, The last bus in the system; $N_{b r}$, Number of branches in the system; $N_{P V}$, Number of PV buses in the system; $P_{l, i}$, Fundamental active power of the linear load at bus $i ; P_{n l, i}$, Fundamental active power of the nonlinear load at bus $i ; q$, Pulse number of the converter; $Q_{l, i}$, Fundamental reactive power of the linear load at bus $i ; Q_{n l, i}$, Fundamental reactive power of the nonlinear load at bus $i ; R_{d g, i}$, Resistance of the linear DG at bus $i ; R_{i}$, Resistance of line/cable $i ; R_{T, i}$, Resistance of transformer $i$; THD, Total harmonic distortion; THD ${ }_{I}$, Total harmonic distortion of current; $\mathrm{THD}_{U}$, Total harmonic distortion of voltage; $U_{\text {nom,i}}$, Amplitude of the nominal operating voltage of load $i ; X_{T, i}$, Leakage reactance of transformer $i ; \underline{Z}_{P V}$, Sensitivity impedance of the PV bus; $\alpha_{l, i}$, Total number of branches starting from bus $i$; $\varepsilon_{U}$, Tolerance limit.


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## Peer Review

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units on voltage profile, power losses, and power quality are carried out. Performance of the proposed method is tested and evaluated on the standard IEEE distribution systems. Simulation results, obtained using the proposed BFS method, are compared with those obtained using the Decoupled Harmonic Power Flow (DHPF) method, Harmonic Analysis module of the ETAP program, and Full Harmonic Solution module of the PCFLO program. It is shown that the BFS method provides effective, robust, and high-quality solutions. Besides that, the proposed method has better computational performance than the commonly used DHPF method, since the bus admittance matrix inverse employed by the DHPF method is not necessary in the solution procedure.

## KEYWORDS

backward/forward sweep, distributed generation, harmonic power flow, power quality, radial distribution system

## 1 | INTRODUCTION

Power flow calculations, commonly referred as to load flow calculations, are among the most used calculations in power systems. They represent the basis of power system analysis and design and are important both for determining the optimal conditions for functioning of the power systems and for planning of further development of the systems. The traditional power flow calculation methods ${ }^{1,2}$ are based on the assumption that the loads in the system are linear and include only fundamental frequency components of voltage and current. However, the increasing presence of nonlinear loads and distributed generation (DG) units connected to the grid via power electronic converters that generate harmonic currents and voltages in the system has imposed a need for developing new methods, as well as adapting existing ones, in order to take the effect of harmonics into account.

Over the last few decades, researchers have proposed a number of methods to solve the power flow problem in the presence of harmonics, also known as the Harmonic Power Flow (HPF) problem. ${ }^{3-21}$ The HPF calculation is more complicated than the traditional power flow calculation and requires extensive computer memory and computing time. The methods vary in terms of data requirements, system condition, modeling complexity, problem formulation, and solution approach. The procedures for analyzing the harmonic problem could be classified into time domain, ${ }^{3}$ frequency domain, ${ }^{4,6-20}$ and hybrid time-frequency domain. ${ }^{21}$ Time domain approaches, such as the electromagnetic transient program (EMTP), ${ }^{3}$ are based on the transient analysis and have great flexibility and high accuracy. They are very useful for the nonlinear device treatment, but the calculation effort necessary to reach the steady state solution could be considerable, and they do not allow the inclusion of the power consumption in the definition of the problem. ${ }^{11}$ Frequency domain methods calculate the frequency response of power systems and reduce the computation time. These methods are a reformulation of the traditional power flow that includes nonlinear devices. The accuracy of the solution depends on the number of harmonics included in the calculation process. Frequency domain methods are the most commonly used methods for solving HPF problems. Hybrid methods combine both domains in order to benefit from the advantages of each domain. They use a combination of frequency domain (to limit the computing time) and time domain (to increase the accuracy) approaches to simulate the power system and nonlinear loads. ${ }^{5}$ Based on their solution approach, HPF methods can also be divided into two categories: coupled and decoupled. Coupled Harmonic Power Flow (CHPF) methods ${ }^{4}$ solve all harmonics simultaneously. They are very accurate, however require exact formulation of nonlinear loads and long computing time. In Decoupled Harmonic Power Flow (DHPF) methods, ${ }^{6-10}$ it is assumed that the coupling between harmonic orders can be rationally neglected, and as a result, the calculation is separately done for each harmonic of interest. The nonlinear loads are presented as decoupled harmonic current sources with associated magnitudes and phase angles. Therefore, these methods require less computational time than CHPF methods and have acceptable accuracy compared with them.

An improved algorithm for the calculation of the HPF in balanced radial distribution systems with laterals is proposed in Safargholi et al. ${ }^{12}$ The algorithm is based on backward and forward sweeps and uses the matrix of nodes connected to different branches of the system. In another research by Peng and Lo, ${ }^{13}$ a Backward/Forward Sweep (BFS)
algorithm for both fundamental and harmonic flow calculations in radial distribution systems with nonlinear DG units is introduced. Galvania et $\mathrm{al}^{14}$ have presented a probabilistic harmonic load flow in unbalanced three-phase distribution systems. In Yang and Le, ${ }^{15}$ a three-phase HPF algorithm for radial distribution systems based on the loop frame of reference is proposed. Teng et al ${ }^{16}$ have presented a three-phase harmonic analysis method for unbalanced distribution systems. In Amini et al,17 a noniterative harmonic load flow method based on the BFS algorithm is proposed and successfully applied to the network reconfiguration problem. By considering the special topology characteristics of radial distribution systems, this method performs harmonic load flow calculations faster than other matrix methods, that is, the admittance matrix method, ${ }^{18}$ admittance summation method, ${ }^{19}$ and direct $\boldsymbol{Z}_{\boldsymbol{B} \boldsymbol{U S}}$ method. ${ }^{20}$ In addition to this, the method is capable of performing the harmonic analysis in radial and weakly meshed distribution systems with capacitor banks and different types of linear/nonlinear loads.

This paper proposes a fast and efficient HPF method for radial distribution systems with DG units. The method is based on the simple practical and approximated component models, frequency domain formulation, and commonly used BFS technique. ${ }^{22}$ Using the BFS technique, the high computational costs for construction and inverse of the bus admittance matrix or impedance matrix, needed for DHPF methods, ${ }^{8,10}$ Newton Raphson-based methods, ${ }^{4,5}$ and other matrix methods, ${ }^{18-20}$ are avoided. The aim was to develop an HPF method with low memory and computation requirements and high performance that can easily be applied to other harmonic problems, such as the problem of the optimal placement and sizing of DG units (converter based and/or nonconverter based) in distribution systems with linear and/or nonlinear loads. The similar HPF methods to the one described herein have been recently published ${ }^{6,7,9}$ but without taking into account the impact of the DG units. The authors of the paper ${ }^{13}$ have introduced an algorithm for harmonic analysis with nonlinear DG units. However, in the HPF calculation, harmonic branch currents are determined directly (noniteratively), thereby neglecting harmonic currents absorbed by linear loads, which makes the calculation simpler and less accurate. Linear loads constitute the main elements of damping, and they may affect the resonance conditions, particularly at higher frequencies. ${ }^{17}$ In this paper, harmonic currents absorbed by linear loads are taken into account when determining harmonic branch currents, which represents a main innovation in relation to the paper. ${ }^{13}$ Linear loads are represented by a parallel combination of a resistor and an inductor. Performance of the proposed method is tested on three standard test systems with nonlinear loads, IEEE 33-bus, IEEE 69-bus, and IEEE 85-bus. To verify the accuracy of the proposed BFS method, the simulation results are compared with those obtained using the DHPF method, ${ }^{8,10}$ Harmonic Analysis module of the ETAP program, ${ }^{23}$ and Full Harmonic Solution module of the PCFLO program. ${ }^{24}$ ETAP uses a decoupled approach for the harmonic load flow formulation, and PCFLO uses a coupled approach. The effects of converter-based and nonconverter-based DG units on voltage profile, power losses, and power quality are carried out.

## 2 | THE PROPOSED METHOD

In the proposed method, the fundamental and harmonic frequency components of voltage and current can be found by repeating the forward and backward sweeps. In the backward sweep, where, starting from the end buses and moving towards to the supply bus, using the first Kirchhoff's Current Law (KCL), the currents at load and generator buses and the branch currents are calculated. In the forward sweep, where, starting in the opposite direction, from the power supply bus and moving forward to the end buses, using the second Kirchhoff's Voltage Law (KVL), the voltage drop on each branch and the voltage at each bus are calculated. More details on the BFS method can be found in previous studies. ${ }^{6,7,9,22}$

## 2.1 | Assumptions

Several practical assumptions have been adopted:

- The distribution system is symmetrical and balanced.
- Voltage at the substation bus is 1.0 p.u.
- The substation voltage does not contain any harmonic component.
- Loads are time invariant.
- At the fundamental frequency, all loads are represented as constant PQ loads, while capacitors are represented as constant impedances.
- At the harmonic frequencies, linear loads are represented by the parallel RL impedance model, and nonlinear loads and DG units are treated as ideal harmonic current sources that inject harmonic currents in the system.
- The harmonic contents (magnitudes and phase angles) of nonlinear loads and DG units are known (measured and estimated).
- The DG units are capable of independently controlling the active power and voltage magnitude and, therefore, operate as PV buses.


## 2.2 | Calculation procedure

The calculation procedure is performed in the following 13 steps:

- Step 1. Initialization of the process for the fundamental frequency $(h=1)$.

The initialization consists of the initial setting of amplitude and phase angle of voltages. Usually, it is assumed that in the initial iteration $(k=0)$, the voltages at all buses are equal to the voltage of the power supply bus $\left(U_{0}^{(1)}\right)$, which is considered as a slack bus:

$$
\begin{equation*}
\left(U_{i}^{(1)}\right)^{(0)}=U_{0}^{(1)} ; \quad\left(\theta_{i}^{(1)}\right)^{(0)}=0 ; \quad i=0,1,2,3, \ldots, N \tag{1}
\end{equation*}
$$

where $V_{\mathrm{i}}^{(0)}$ and $\left(\theta_{i}^{(1)}\right)^{(0)}$, respectively, are the initial values of the fundamental voltage amplitude and phase angle at bus $i$ and $N$ is the last bus in the system.

- Step 2. Iteration updating.

The iteration index is updated $k=k+1$.

- Step 3. Performing the power flow calculation for the fundamental frequency.

The calculation of unknown voltages and currents is carried out using a standard BFS method. ${ }^{22}$ Each iteration consists of the following two steps:
a. Backward sweep—current calculation.

In this step, the calculation of branch currents is performed, starting from the most distant buses of the power supply bus, that is, from the end buses, and moving to the supply bus. According to the equivalent scheme in Figure 1, the fundamental current through branch $i$ at iteration $k\left(\underline{J}_{i}^{(1)}\right)^{(k)}$ can be found by applying the first KCL:


FIGURE 1 The current distribution in the $i$-th bus of the system

$$
\begin{equation*}
\left(\underline{J}_{i}^{(1)}\right)^{(k)}=\left(I_{l, i}^{(1)}\right)^{(k)}+\left(\underline{I}_{n, i}^{(1)}\right)^{(k)}-\left(\underline{I}_{d g, i}^{(1)}\right)^{(k)}+\underline{Y}_{s h, i}^{(1)}\left(\underline{U}_{i}^{(1)}\right)^{(k-1)}+\sum_{l \in \alpha_{l, i}}\left(\underline{J}_{l}^{(1)}\right)^{(k)} ; i=N, N-1, \ldots, 0, \tag{2}
\end{equation*}
$$

where $\left(I_{l, i}^{(1)}\right)^{(k)},\left(I_{n l, i}^{(1)}\right)^{(k)}$, and $\left(I_{d g, i}^{(1)}\right)^{(k)}$ are the complex currents of the linear load at bus $i$, nonlinear load at bus $i$, and DG at bus $i$, respectively; $\underline{Y}_{s h, i}^{(1)}$ is the sum of the shunt admittance of elements incident to bus $i ;\left(\underline{U}_{i}^{(1)}\right)^{(k-1)}$ is the complex voltage at bus $i$ from the previous $(k-1)$ iteration; $\alpha_{l, i}$ is the total number of branches starting from bus $i$.

At the fundamental frequency, the loads can be modeled as constant power, constant current, constant impedance, or any combination of these models. The current injections of the linear and nonlinear loads modeled as constant power loads at the $k$-th iteration are

$$
\begin{align*}
& \left(I_{l, i}^{(1)}\right)^{(k)}=\left(\frac{P_{l, i}+j Q_{l, i}}{\left(\underline{U}_{i}^{(1)}\right)^{(k-1)}}\right)^{*},  \tag{3}\\
& \left(I_{n l, i}^{(1)}\right)^{(k)}=\left(\frac{P_{n l, i}+j Q_{n l, i}}{\left(\underline{U}_{i}^{(1)}\right)^{(k-1)}}\right)^{*}, \tag{4}
\end{align*}
$$

where $P_{l, i}$ and $Q_{l, i}$ are the fundamental active and reactive powers of the linear load at bus $i$, respectively, while $P_{n l, i}$ and $Q_{n l, i}$ are the fundamental active and reactive powers of the nonlinear load at bus $i$, respectively.

Depending on the applied technologies, the primary energy of DG units may be injected into a distribution system via either a synchronous or asynchronous electric machine, which is directly connected to the grid or only via a power electronic interface. If the electric machine is directly connected to the grid, its operation determines the model of the DG unit (PQ bus or PV bus). In other cases, the characteristics of the interface control circuit determine the DG unit model. As a general rule, in case when the control circuit of the converter is designed to control active power and voltage independently, the DG unit model shall be as a PV bus, and when it is designed to control active and reactive power independently, the DG unit model shall be as a PQ bus. ${ }^{25}$ When the DG is modeled as a PQ bus, the expression for the current injection of the DG at bus $i$ is

$$
\begin{equation*}
\left(\underline{I}_{d g, i}^{(1)}\right)^{(k)}=\left(\frac{P_{d g, i}^{s p}+j Q_{d g, i}^{s p}}{\left(\underline{U}_{i}^{(1)}\right)^{(k-1)}}\right)^{*}, \tag{5}
\end{equation*}
$$

where $P_{d g, i}^{s p}$ and $Q_{d g, i}^{s p}$ are the specified active and reactive powers of the DG at bus $i$, respectively.
On the other hand, when the DG operates as a PV bus, the expression for the current injection becomes

$$
\begin{equation*}
\left(I_{d g, i}^{(1)}\right)^{(k)}=\left(\frac{P_{d g, i}^{s p}+j Q_{d g, i}^{(k-1)}}{\left(\underline{U}_{i}^{(1)}\right)^{(k-1)}}\right)^{*} . \tag{6}
\end{equation*}
$$

At this point, it is necessary to calculate the reactive power of DG. This power can be calculated according to the following equation:

$$
\begin{equation*}
Q_{d g}^{(k)}=Q_{d g}^{(k-1)}+\operatorname{Im}\left\{\underline{U}_{d g}^{s p}\left[\underline{Z}_{P V}^{-1}\left(\underline{U}_{d g}^{s p}-\underline{U}_{d g}^{(k)}\right)\right]^{*}\right\} \tag{7}
\end{equation*}
$$

where $\underline{U}_{d g}^{s p}$ is the specified voltage at the PV bus, $\underline{U}_{d g}^{(k)}$ is the voltage at the PV bus obtained at iteration $k$, and $\underline{Z}_{P V}$ is the complex sensitivity impedance of the PV bus, which is calculated by summing the impedances of the branches between PV bus and power supply bus. If there is more than one DG in the system, then the above equation takes the following vector form:

$$
\begin{equation*}
\mathbf{Q}_{d g}^{(k)}=\mathbf{Q}_{d g}^{(k-1)}+\operatorname{Im}\left\{\underline{\mathbf{U}}_{d g}^{s p}\left[\underline{\mathbf{Z}}_{P V}^{-1}\left(\underline{\mathbf{U}}_{d g}^{s p}-\underline{\mathbf{U}}_{d g}^{(k)}\right)\right]^{*}\right\}, \tag{8}
\end{equation*}
$$

where $\mathbf{Q}_{d g}=\left[Q_{d g, 1}, Q_{d g, 2}, \ldots, Q_{d g, N_{P V}}\right]^{\mathrm{T}}$ is the reactive power injection vector of PV buses, $\underline{\mathbf{U}}_{d g}^{s p}=\left[U_{d g, 1}^{s p}, U_{d g, 2}^{s p}, \ldots, U_{d g, N_{P V}}^{s p}\right]^{\mathrm{T}}$ is the specified voltage vector of PV buses, $\underline{U}_{d g}=\left[U_{d g, 1}, U_{d g, 2}, \ldots, U_{d g, N_{P V}}\right]^{\mathrm{T}}$ is the vector of calculated voltages of PV buses, $\underline{Z}_{P V}$ is the complex sensitivity impedance matrix of PV buses, and $N_{P V}$ is the number of PV buses. There is the possibility that the calculated reactive power of a DG is outside the given limits, $Q_{d g}^{\min }<Q_{d g}<Q_{d g}^{\max }$, in this case, the reactive power of the generator is set to the specified limit, and in the further calculations, this bus is treated as a PQ bus with the specified active and reactive powers.
b. Forward sweep-voltage calculation

Starting from the power supply bus and moving forward to the end buses, new bus voltage values are determined applying the second KVL:

$$
\begin{equation*}
\left(\underline{U}_{i}^{(1)}\right)^{(k)}=\left(\underline{U}_{0}^{(1)}\right)^{(k)}-\underline{Z}_{i}^{(1)} \cdot\left(\underline{J}_{i}^{(1)}\right)^{(k)}, \tag{9}
\end{equation*}
$$

where $\underline{Z}_{i}^{(1)}$ is the fundamental impedance of line $i$.

- Step 4. Checking the termination criterion for the fundamental frequency.

Steps 3 a and 3 b are iteratively executed while the maximum difference between bus voltages of two successive iterations is less than an acceptable tolerance limit,

$$
\begin{equation*}
\varepsilon_{U} \geq \max \left|\left(U_{i}^{(1)}\right)^{(k)}-\left(U_{i}^{(1)}\right)^{(k-1)}\right|, \tag{10}
\end{equation*}
$$

or the maximum number of iterations is reached, $k=k_{\text {max }}$.

- Step 5. Modification of distribution system elements at harmonic frequencies $(h \geq 2)$.

At harmonic frequencies ( $h \geq 2$ ), a distribution system is modeled as a combination of passive elements and harmonic current sources. Instead of using the very accurate models, some practical and approximated models of previous studies ${ }^{5-10,26-28}$ are used in this paper.

### 2.2.1 | Distribution lines and cables

Distribution lines and cables can be represented by the lumped parameter elements using a $\pi$-connection. If the skin and proximity effects are neglected at higher frequencies, the longitudinal and shunt parameters $\left(\underline{Y}_{i}^{(h)}\right.$ and $\underline{Y}_{s h i,}^{(h)}$ ) of line/ cable $i$ are ${ }^{10}$

$$
\begin{gather*}
\underline{Y}_{i}^{(h)}=\frac{1}{R_{i}+j 2 \pi f h L_{i}},  \tag{11}\\
\underline{Y}_{s h, i}^{(h)}=j 2 \pi f h C_{i} \tag{12}
\end{gather*}
$$

where $R_{i}, L_{i}$, and $C_{i}$ represent the resistance, inductance, and capacitance of line/cable $i$, respectively; $f$ is the fundamental frequency (ie, $f=50 \mathrm{~Hz}$ ), and $h$ is the harmonic order. The skin effect can be included in (11) by modifying the resistive part of the line admittance as follows ${ }^{10,26}$ :

$$
\begin{align*}
R_{i}^{(h)} & =R_{i}\left(1+\frac{0.646 h^{2}}{192+0.518 h^{2}}\right)  \tag{13}\\
R_{i}^{(h)} & =R_{i}(0.187+0.532 \sqrt{h}) \tag{14}
\end{align*}
$$

Equation (13) is for overhead lines and Equation (14) for power cables.

### 2.2.2 | Transformers

Complete representation of transformers, including capacitances, is not practical and cannot be justified for harmonic frequencies. For a practical analysis of the harmonics can be used a simple model expressed by the following equation ${ }^{7}$ :

$$
\begin{equation*}
\underline{Y}_{T, i}^{(h)}=\frac{1}{R_{T, i}+j h X_{T, i}}, \tag{15}
\end{equation*}
$$

where $R_{T, i}$ and $X_{T, i}$, respectively, are the resistance and leakage reactance of transformer $i$, calculated at the fundamental frequency.

### 2.2.3 | Linear and nonlinear loads

Different types of linear load models at harmonic frequencies are recommended. ${ }^{26,27}$ The choice of the load model to use depends on the nature of the load and on the available information. The generalized model is suggested for linear loads, which is composed of a resistance in parallel with an inductance. If the skin effect is neglected at higher frequencies, the admittance of the linear load connected at bus $i\left(\underline{Y}_{l, i}^{(h)}\right)$ is $^{26}$

$$
\begin{equation*}
\underline{Y}_{l, i}^{(h)}=\frac{P_{l, i}}{U_{n o m, i}^{2}}-j \frac{Q_{l, i}}{h U_{n o m, i}^{2}}, \tag{16}
\end{equation*}
$$

where $U_{\text {nom, } i}$ is the amplitude of the nominal operating voltage of load $i$, which is 1 p.u.
Nonlinear loads are treated as decoupled harmonic current sources that inject harmonic currents into the system. Harmonics generated by a converter of any pulse number can be expressed as ${ }^{27}$

$$
\begin{equation*}
h=n q \pm 1 \tag{17}
\end{equation*}
$$

where $n$ is any integer ( $1,2, \ldots$, etc.) and $q$ is the pulse number of the converter ( 6 in the case of a six-pulse converter). According to Equation (17), it is obvious that the characteristic harmonics for a three phase, six-pulse converter are all odd harmonics except triplens (5th, 7th, 11th, etc).

The $h$-th harmonic current injected by the nonlinear load at bus $i\left(I_{n l, i}^{(h)}\right)$ is defined by ${ }^{8}$

$$
\begin{equation*}
I_{n l, i}^{(h)}=C(h) I_{n l, i}^{(1)}, \tag{18}
\end{equation*}
$$

where $I_{n, i}^{(1)}$ is the amplitude of the fundamental current injected by the nonlinear load at bus $i$, and $C(h)$ is the ratio of the $h$-th harmonic current to its fundamental value.

The phase angle of the harmonic current injected by the nonlinear load at bus $i\left(\theta_{n l, i}^{(h)}\right)$ can be expressed as ${ }^{28}$

$$
\begin{equation*}
\theta_{n l, i}^{(h)}=\theta_{n l, i}^{(h-\text { spectrum })}+h \theta_{n l, i}^{(1)}+(h+1) \frac{\pi}{2}, \tag{19}
\end{equation*}
$$

where $\theta_{n, l}^{(1)}$ is the phase angle obtained from the power flow solution for the fundamental frequency current component and $\theta_{n, i}^{(h-\text { spectrum })}$ is the typical phase angle of the harmonic source current spectrum.

### 2.2.4 | DG units

In terms of the interfacing devices to the grid, DG units can be grouped into two categories ${ }^{29}$ : converter-based DG, such as PV systems, wind turbine generators, fuel cells, and microturbines, and nonconverter-based DG, like minihydro synchronous generators and induction generators. From the perspective of harmonic modeling and simulation, it may be assumed that the nonconverter-based DGs are linear, produce no harmonics, and can be represented by shunt equivalent admittance. On the other hand, the converter-based DGs are nonlinear and can be viewed as negative nonlinear loads that inject harmonic currents into the system. For the purpose of determining the system harmonic admittances, a linear DG unit can be modeled as a series combination of a resistance and an inductive reactance, ${ }^{26}$ that is,

$$
\begin{equation*}
\underline{Y}_{d g, i}^{(h)}=\frac{1}{\sqrt{h} R_{d g, i}+j h X_{d g, i}^{\prime \prime}}, \tag{20}
\end{equation*}
$$

where $R_{d g, i}$ and $X_{d g, i}^{\prime \prime}$ are the resistance and subtransient reactance of generator $i$, respectively.

- Step 6. Initialization of the process for harmonic frequencies ( $h \geq 2$ ).

If the substation voltage does not contain any harmonic component, then it can be assumed that in the initial iteration ( $k=0$ ), the voltages at all buses are equal to zero, that is,

$$
\begin{equation*}
\left(U_{i}^{(h)}\right)^{(0)}=0 ; \quad\left(\theta_{i}^{(h)}\right)^{(0)}=0 ; \quad i=0,1,2,3, \ldots, \mathrm{~N} . \tag{21}
\end{equation*}
$$

- Step 7. Iteration updating.

The iteration index is updated $k=k+1$.

- Step 8. Performing the power flow calculation for harmonic frequencies.

For the calculation of the unknown harmonic frequency components of voltage and current, a similar iterative procedure is used as that described in step 3.
a. Backward sweep-current calculation.

As previously mentioned, DGs that use power electronic converters to connect to a distribution system can be presented as generators of higher current harmonics. In this case, the iterative procedure used here differs from the one
described above in step 3a in that the harmonic current injections from nonlinear loads, as well as DG units, remain constant during the entire process, that is,

$$
\begin{gather*}
\left(\underline{J}_{i}^{(h)}\right)^{(k)}=\left(\underline{I}_{l, i}^{(h)}\right)^{(k)}-\underline{I}_{n l, i}^{(h)}+\underline{I}_{d g, i}^{(h)}+\underline{Y}_{s h, i}^{(h)} \cdot\left(\underline{U}_{i}^{(h)}\right)^{(k-1)}+\sum_{l \in \alpha_{l, i}}\left(\underline{J}_{l}^{(h)}\right)^{(k)} ; i=N, N-1, \ldots, 0  \tag{22}\\
l \neq i
\end{gather*}
$$

where $\left(\underline{J}_{i}^{(h)}\right)^{(k)}$ is the $h$-th harmonic current through branch $i$ at iteration $k ;\left(\underline{I}_{l, i}^{(h)}\right)^{(k)}$ is the $h$-th harmonic current of linear load $i$ at iteration $k ; \underline{I}_{n l, i}^{(h)}$ is the $h$-th harmonic current injected by the nonlinear load into bus $i ; \underline{I}_{d g, i}^{(h)}$ is the $h$-th harmonic current injected by the nonlinear DG into bus $i$, calculated in the same manner as the current of the nonlinear load; $\underline{Y}_{s h, i}^{(h)}$ is the modified shunt admittance of the elements in bus $i$ at harmonic $h$, and $\left(\underline{U}_{i}^{(h)}\right)^{(k-1)}$ is the $h$-th harmonic component of the voltage at bus $i$ from the previous $(k-1)$ iteration.

If the system contains only linear DG units, which are directly connected to the grid, then in (22), the $h$-th harmonic current of linear DG $i$ at iteration $k$ should be calculated according to the following formula:

$$
\begin{equation*}
\left(\underline{I}_{d g, i}^{(h)}\right)^{(k)}=\underline{Y}_{d g, i}^{(h)} \cdot\left(\underline{U}_{i}^{(h)}\right)^{(k-1)} \tag{24}
\end{equation*}
$$

b. Forward sweep-voltage calculation.

In this step, starting from the power supply bus in which the harmonic voltage is equal to 0 and moving forward to the end buses, harmonic voltages at all buses are determined:

$$
\begin{equation*}
\left(\underline{U}_{i}^{(h)}\right)^{(k)}=\left(\underline{U}_{0}^{(h)}\right)^{(k)}-\underline{Z}_{i}^{(h)} \cdot\left(\underline{J}_{i}^{(h)}\right)^{(k)}=-\underline{Z}_{i}^{(h)} \cdot\left(\underline{J}_{i}^{(h)}\right)^{(k)}, \tag{25}
\end{equation*}
$$

where $\underline{Z}_{i}^{(h)}$ is the impedance of line $i$ at harmonic $h$.

- Step 9. Checking the termination criterion for each harmonic frequency.

The described procedure in steps 8 a and 8 b is repeated for each frequency of interest until the termination criterion is satisfied. It is the same as one in step 4.

- Step 10. Checking whether the specified highest harmonic of interest is reached.

If the highest harmonic of interest is reached $\left(h=h_{\max }\right)$, then go to the next step; otherwise, set harmonic order $h=h+1$ and return to step 5 .

- Step 11. Calculation of the power flows and power losses in the system.

At the end of the calculation, the proposed technique of power flow can calculate the power flows in branches $\left(\underline{S}_{i}^{(h)}\right)$, power losses in branches $\left(\underline{S}_{l o s s, i}^{(h)}\right)$, and total power losses of the system for all harmonics $\left(\underline{S}_{l o s s}^{(h)}\right)$ :

$$
\begin{align*}
& \underline{S}_{i}^{(h)}=\underline{U}_{i}^{(h)} \cdot\left(\underline{J}_{i}^{(h)}\right)^{*}  \tag{26}\\
& \underline{S}_{l o s s, i}^{(h)}=\underline{Z}_{i}^{(h)} \cdot\left(J_{i}^{(h)}\right)^{2} \tag{27}
\end{align*}
$$

$$
\begin{equation*}
\underline{S}_{l o s s}=\sum_{h=1}^{h_{\max }}\left(\sum_{i=1}^{N_{b r}} \underline{S}_{l o s s, i}^{(h)}\right) \tag{28}
\end{equation*}
$$

where $N_{b r}$ denotes the total number of branches in the system.

- Step 12. Storing obtained results.

The results of the calculation are the voltages and currents of the fundamental and higher harmonics, power flows through branches of the system, and power losses in the system elements.

- Step 13. Calculation of the values of harmonic indicators.

The most common harmonic indicator, which widely used to describe power quality issues in transmission and distribution systems, is the total harmonic distortion (THD). At any bus $i$, THD is defined for voltage and current signals, respectively, as follows:

$$
\begin{equation*}
\mathrm{THD}_{U, i}=\frac{\sqrt{\sum_{h=2}^{h_{\max }}\left(U_{i}^{(h)}\right)^{2}}}{U_{i}^{(1)}} \tag{29}
\end{equation*}
$$



FIGURE 2 Single-line diagram of the IEEE 33-bus test system

| Harmonic <br> order | PWM adjustable-speed drive |  |  | Six-pulse converter |  |
| :--- | :---: | :---: | :---: | :---: | :--- |
|  | Magnitude, $\%$ | Phase angle, ${ }^{\circ}$ |  | Magnitude, $\%$ | Phase angle, ${ }^{\circ}$ |
| 5 | 100 | 0 |  | 100 | 0 |
| 7 | 72.8 | -135 | 20 | 0 |  |
| 11 | 47.5 | 69 | 14.3 | 0 |  |
| 13 | 41.2 | -62 | 9.1 | 0 |  |
| 17 | 14.2 | 139 | 7.7 | 0 |  |
| 19 | 9.7 | 9 | 5.9 | 0 |  |
| 23 | 1.5 | -155 | 5.3 | 0 |  |
| 25 | 2.5 | -158 | 4.3 | 0 |  |
| 29 | 0 | 98 | 4 | 0 |  |
| 31 | 0 | 0 | 3.4 | 0 |  |

TABLE 1 The harmonic spectrums of nonlinear loads and DG unit

Abbreviation: DG, distributed generation.

$$
\begin{equation*}
\mathrm{THD}_{I, i}=\frac{\sqrt{\sum_{h=2}^{h_{\max }}\left(I_{i}^{(h)}\right)^{2}}}{I_{i}^{(1)}} \tag{30}
\end{equation*}
$$

According to the IEEE-519 standard, ${ }^{30}$ at the point of common coupling in distribution systems below 69 kV , the level of the $\mathrm{THD}_{U}$ must be lower than $5 \%$.

TABLE 2 RMS voltages and THD $_{U}$ values of the IEEE 33-bus test system for case 1

| Bus | $V_{\text {RMS }}$, p.u. |  |  |  |  | $\mathrm{THD}_{U}, \%$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BFS | DHPF | ETAP | PCFLO | Heydari et $\mathbf{a l}^{32}$ | BFS | DHPF | ETAP | PCFLO | $\text { Heydari et al }{ }^{32}$ |
| 0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.9952 | 0.9952 | 0.9952 | 0.9952 | 0.9952 | 0.2292 | 0.2292 | 0.2335 | 0.2300 | 0.2300 |
| 2 | 0.9717 | 0.9717 | 0.9717 | 0.9717 | 0.9717 | 1.4898 | 1.4898 | 1.4974 | 1.4900 | 1.5100 |
| 3 | 0.9574 | 0.9574 | 0.9574 | 0.9573 | 0.9573 | 2.4658 | 2.4659 | 2.4731 | 2.4600 | 2.5000 |
| 4 | 0.9429 | 0.9429 | 0.9430 | 0.9429 | 0.9429 | 3.5139 | 3.5140 | 3.5214 | 3.5100 | 3.5500 |
| 5 | 0.9077 | 0.9077 | 0.9077 | 0.9076 | 0.9077 | 7.4365 | 7.4367 | 7.4440 | 7.4300 | 7.5000 |
| 6 | 0.9039 | 0.9039 | 0.9039 | 0.9039 | 0.904 | 7.3566 | 7.3569 | 7.3640 | 7.3500 | 7.5300 |
| 7 | 0.8988 | 0.8988 | 0.8988 | 0.8988 | 0.8989 | 7.3426 | 7.3428 | 7.3499 | 7.3400 | 7.5700 |
| 8 | 0.8922 | 0.8922 | 0.8922 | 0.8921 | 0.8924 | 7.2952 | 7.2955 | 7.3025 | 7.2900 | 7.6300 |
| 9 | 0.8860 | 0.8860 | 0.8860 | 0.8860 | 0.8863 | 7.2658 | 7.2660 | 7.2730 | 7.2600 | 7.6800 |
| 10 | 0.8851 | 0.8851 | 0.8851 | 0.8851 | 0.8854 | 7.2648 | 7.2650 | 7.2720 | 7.2600 | 7.6900 |
| 11 | 0.8835 | 0.8835 | 0.8835 | 0.8835 | 0.8838 | 7.2630 | 7.2631 | 7.2701 | 7.2600 | 7.7000 |
| 12 | 0.8770 | 0.8770 | 0.8770 | 0.8770 | 0.8774 | 7.2417 | 7.2417 | 7.2487 | 7.2400 | 7.7600 |
| 13 | 0.8747 | 0.8747 | 0.8747 | 0.8746 | 0.875 | 7.2362 | 7.2361 | 7.2432 | 7.2300 | 7.7800 |
| 14 | 0.8732 | 0.8732 | 0.8732 | 0.8731 | 0.8735 | 7.2351 | 7.2350 | 7.2421 | 7.2300 | 7.8000 |
| 15 | 0.8717 | 0.8717 | 0.8717 | 0.8717 | 0.8721 | 7.2353 | 7.2352 | 7.2423 | 7.2300 | 7.8100 |
| 16 | 0.8696 | 0.8696 | 0.8696 | 0.8695 | 0.8699 | 7.2359 | 7.2357 | 7.2429 | 7.2300 | 7.8300 |
| 17 | 0.8690 | 0.8690 | 0.8689 | 0.8689 | 0.8693 | 7.2371 | 7.2369 | 7.2442 | 7.2300 | 7.8300 |
| 18 | 0.9947 | 0.9947 | 0.9947 | 0.9947 | 0.9947 | 0.2292 | 0.2292 | 0.2335 | 0.2300 | 0.2300 |
| 19 | 0.9911 | 0.9911 | 0.9911 | 0.9911 | 0.9911 | 0.2289 | 0.2289 | 0.2344 | 0.2300 | 0.2300 |
| 20 | 0.9904 | 0.9904 | 0.9904 | 0.9904 | 0.9904 | 0.2289 | 0.2289 | 0.2355 | 0.2300 | 0.2300 |
| 21 | 0.9898 | 0.9898 | 0.9898 | 0.9898 | 0.9897 | 0.2289 | 0.2289 | 0.2358 | 0.2300 | 0.2300 |
| 22 | 0.9681 | 0.9681 | 0.9681 | 0.9680 | 0.9681 | 1.4858 | 1.4859 | 1.4929 | 1.4900 | 1.5200 |
| 23 | 0.9613 | 0.9613 | 0.9613 | 0.9613 | 0.9613 | 1.4816 | 1.4816 | 1.4886 | 1.4800 | 1.5300 |
| 24 | 0.9580 | 0.9580 | 0.9580 | 0.9579 | 0.9579 | 1.4812 | 1.4812 | 1.4891 | 1.4800 | 1.5300 |
| 25 | 0.9040 | 0.9040 | 0.9040 | 0.9039 | 0.904 | 7.7514 | 7.7516 | 7.7587 | 7.7500 | 7.8200 |
| 26 | 0.8990 | 0.8990 | 0.8990 | 0.8989 | 0.899 | 8.1964 | 8.1966 | 8.2038 | 8.1900 | 8.2600 |
| 27 | 0.8868 | 0.8868 | 0.8868 | 0.8867 | 0.8869 | 8.1325 | 8.1327 | 8.1399 | 8.1300 | 8.3800 |
| 28 | 0.8780 | 0.8780 | 0.8780 | 0.8780 | 0.8783 | 8.1169 | 8.1171 | 8.1243 | 8.1200 | 8.4600 |
| 29 | 0.8743 | 0.8743 | 0.8743 | 0.8742 | 0.8745 | 8.1188 | 8.1190 | 8.1261 | 8.1200 | 8.5000 |
| 30 | 0.8698 | 0.8698 | 0.8698 | 0.8698 | 0.8701 | 8.1176 | 8.1178 | 8.1249 | 8.1200 | 8.5400 |
| 31 | 0.8689 | 0.8689 | 0.8689 | 0.8688 | 0.8692 | 8.1193 | 8.1194 | 8.1265 | 8.1200 | 8.5500 |
| 32 | 0.8686 | 0.8686 | 0.8686 | 0.8685 | 0.8689 | 8.1200 | 8.1201 | 8.1272 | 8.1200 | 8.5500 |

[^1]| Bus | $V_{\text {RMS }}$, p.u. |  |  |  | $\mathrm{THD}_{U}, \%$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BFS | DHPF | ETAP | PCFLO | BFS | DHPF | ETAP | PCFLO |
| 0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 0.0043 | 0.0000 | 0.0000 |
| 1 | 0.9970 | 0.9970 | 0.9970 | 0.9970 | 0.1316 | 0.1358 | 0.1374 | 0.1300 |
| 2 | 0.9830 | 0.9830 | 0.9830 | 0.9829 | 0.8470 | 0.8512 | 0.8535 | 0.8500 |
| 3 | 0.9756 | 0.9756 | 0.9755 | 0.9755 | 1.3903 | 1.3944 | 1.3974 | 1.3900 |
| 4 | 0.9682 | 0.9682 | 0.9682 | 0.9682 | 1.9647 | 1.9688 | 1.9725 | 1.9600 |
| 5 | 0.9504 | 0.9504 | 0.9504 | 0.9504 | 4.0492 | 4.0532 | 4.0609 | 4.0500 |
| 6 | 0.9474 | 0.9474 | 0.9474 | 0.9474 | 5.1407 | 5.1446 | 5.1536 | 5.1400 |
| 7 | 0.9426 | 0.9426 | 0.9426 | 0.9426 | 5.2076 | 5.2114 | 5.2204 | 5.2000 |
| 8 | 0.9364 | 0.9364 | 0.9364 | 0.9364 | 5.3587 | 5.3624 | 5.3710 | 5.3600 |
| 9 | 0.9307 | 0.9307 | 0.9307 | 0.9306 | 5.5203 | 5.5239 | 5.5319 | 5.5200 |
| 10 | 0.9298 | 0.9298 | 0.9298 | 0.9298 | 5.5405 | 5.5443 | 5.5522 | 5.5400 |
| 11 | 0.9283 | 0.9283 | 0.9283 | 0.9283 | 5.5792 | 5.5831 | 5.5909 | 5.5800 |
| 12 | 0.9224 | 0.9224 | 0.9223 | 0.9223 | 5.8474 | 5.8514 | 5.8584 | 5.8400 |
| 13 | 0.9202 | 0.9202 | 0.9202 | 0.9201 | 6.0065 | 6.0107 | 6.0172 | 6.0000 |
| 14 | 0.9188 | 0.9188 | 0.9188 | 0.9188 | 6.1403 | 6.1449 | 6.1510 | 6.1400 |
| 15 | 0.9176 | 0.9176 | 0.9175 | 0.9175 | 6.2925 | 6.2975 | 6.3032 | 6.2900 |
| 16 | 0.9158 | 0.9158 | 0.9158 | 0.9157 | 6.7652 | 6.7704 | 6.7749 | 6.7600 |
| 17 | 0.9152 | 0.9152 | 0.9152 | 0.9151 | 6.7652 | 6.7710 | 6.7755 | 6.7700 |
| 18 | 0.9965 | 0.9965 | 0.9965 | 0.9965 | 0.1316 | 0.1358 | 0.1367 | 0.1300 |
| 19 | 0.9929 | 0.9929 | 0.9929 | 0.9929 | 0.1316 | 0.1358 | 0.1367 | 0.1300 |
| 20 | 0.9922 | 0.9922 | 0.9922 | 0.9922 | 0.1316 | 0.1358 | 0.1361 | 0.1300 |
| 21 | 0.9916 | 0.9916 | 0.9916 | 0.9916 | 0.1316 | 0.1358 | 0.1363 | 0.1300 |
| 22 | 0.9794 | 0.9794 | 0.9794 | 0.9794 | 0.8465 | 0.8506 | 0.8521 | 0.8500 |
| 23 | 0.9727 | 0.9727 | 0.9727 | 0.9727 | 0.8459 | 0.8501 | 0.8521 | 0.8500 |
| 24 | 0.9694 | 0.9694 | 0.9694 | 0.9694 | 0.8460 | 0.8501 | 0.8520 | 0.8500 |
| 25 | 0.9486 | 0.9486 | 0.9485 | 0.9485 | 4.1775 | 4.1814 | 4.1893 | 4.1800 |
| 26 | 0.9461 | 0.9461 | 0.9461 | 0.9460 | 4.3577 | 4.3617 | 4.3701 | 4.3600 |
| 27 | 0.9351 | 0.9351 | 0.9351 | 0.9351 | 5.4932 | 5.4971 | 5.5086 | 5.4900 |
| 28 | 0.9274 | 0.9274 | 0.9274 | 0.9273 | 6.3653 | 6.3691 | 6.3832 | 6.3600 |
| 29 | 0.9240 | 0.9240 | 0.9240 | 0.9240 | 6.7051 | 6.7089 | 6.7240 | 6.7000 |
| 30 | 0.9207 | 0.9207 | 0.9206 | 0.9206 | 7.9174 | 7.9212 | 7.9397 | 7.9200 |
| 31 | 0.9197 | 0.9198 | 0.9197 | 0.9197 | 7.9183 | 7.9223 | 7.9408 | 7.9200 |
| 32 | 0.9195 | 0.9195 | 0.9194 | 0.9194 | 7.9185 | 7.9227 | 7.9412 | 7.9200 |

Abbreviations: BFS, Backward/Forward Sweep; DHPF, Decoupled Harmonic Power Flow; THD $_{U}$, Total harmonic distortion of voltage.

$$
0 x+0.0
$$

TABLE 3 RMS voltages and $\mathrm{THD}_{U}$ values of the IEEE 33-bus test system for case 2

$$
5
$$

## 3 | RESULTS AND DISCUSSION

The proposed method for the HPF calculation in radial distribution systems was evaluated using the IEEE 33-bus, ${ }^{31,32}$ IEEE 69-bus, ${ }^{33}$ and IEEE 85 -bus ${ }^{34}$ radial distribution test systems. The first system is used to establish the accuracy of the BFS method, while the second and third ones are used to compare the execution time of the BFS method with the execution time of the DHPF method. The single line diagram of the IEEE 33-bus system is shown in Figure 2. The base voltage for this system is 12.66 kV , and base power is 10 MVA . The BFS method was implemented in the MATLAB computing environment. All simulation data were obtained using a PC with a CPU at 2.70 GHz and 8.0 GB RAM.

TABLE 4 RMS voltages and THD $_{U}$ values of the IEEE 33-bus test system for case 3

| Bus | $V_{\text {RMS }}$, p.u. |  |  |  | $\mathrm{THD}_{U}, \%$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BFS | DHPF | ETAP | PCFLO | BFS | DHPF | ETAP | PCFLO |
| 0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.9986 | 0.9986 | 0.9986 | 0.9986 | 0.1108 | 0.1109 | 0.1156 | 0.1100 |
| 2 | 0.9928 | 0.9928 | 0.9928 | 0.9928 | 0.7073 | 0.7073 | 0.7103 | 0.7100 |
| 3 | 0.9916 | 0.9916 | 0.9916 | 0.9916 | 1.1536 | 1.1536 | 1.1564 | 1.1500 |
| 4 | 0.9906 | 0.9906 | 0.9906 | 0.9906 | 1.6195 | 1.6194 | 1.6223 | 1.6200 |
| 5 | 0.9884 | 0.9884 | 0.9884 | 0.9884 | 3.2829 | 3.2828 | 3.2850 | 3.2800 |
| 6 | 0.9854 | 0.9854 | 0.9854 | 0.9854 | 4.3001 | 4.2998 | 4.3011 | 4.3000 |
| 7 | 0.9808 | 0.9808 | 0.9808 | 0.9808 | 4.3628 | 4.3625 | 4.3630 | 4.3600 |
| 8 | 0.9749 | 0.9749 | 0.9749 | 0.9749 | 4.5110 | 4.5107 | 4.5084 | 4.5100 |
| 9 | 0.9694 | 0.9694 | 0.9694 | 0.9693 | 4.6698 | 4.6695 | 4.6647 | 4.6700 |
| 10 | 0.9685 | 0.9685 | 0.9685 | 0.9685 | 4.6891 | 4.6888 | 4.6837 | 4.6900 |
| 11 | 0.9671 | 0.9671 | 0.9671 | 0.9671 | 4.7259 | 4.7259 | 4.7204 | 4.7200 |
| 12 | 0.9614 | 0.9614 | 0.9614 | 0.9614 | 4.9914 | 4.9915 | 4.9821 | 4.9900 |
| 13 | 0.9593 | 0.9593 | 0.9593 | 0.9593 | 5.1512 | 5.1515 | 5.1398 | 5.1500 |
| 14 | 0.9580 | 0.9580 | 0.9580 | 0.9580 | 5.2842 | 5.2848 | 5.2716 | 5.2800 |
| 15 | 0.9568 | 0.9568 | 0.9567 | 0.9567 | 5.4343 | 5.4354 | 5.4206 | 5.4300 |
| 16 | 0.9551 | 0.9551 | 0.9551 | 0.9550 | 5.9033 | 5.9048 | 5.8852 | 5.9000 |
| 17 | 0.9545 | 0.9545 | 0.9545 | 0.9545 | 5.9029 | 5.9051 | 5.8856 | 5.9000 |
| 18 | 0.9981 | 0.9981 | 0.9981 | 0.9981 | 0.1108 | 0.1108 | 0.1142 | 0.1100 |
| 19 | 0.9945 | 0.9945 | 0.9945 | 0.9945 | 0.1108 | 0.1108 | 0.1139 | 0.1100 |
| 20 | 0.9938 | 0.9938 | 0.9938 | 0.9938 | 0.1108 | 0.1108 | 0.1156 | 0.1100 |
| 21 | 0.9932 | 0.9932 | 0.9932 | 0.9931 | 0.1108 | 0.1108 | 0.1157 | 0.1100 |
| 22 | 0.9893 | 0.9893 | 0.9893 | 0.9893 | 0.7067 | 0.7067 | 0.7098 | 0.7100 |
| 23 | 0.9827 | 0.9827 | 0.9827 | 0.9827 | 0.7062 | 0.7062 | 0.7096 | 0.7100 |
| 24 | 0.9794 | 0.9794 | 0.9794 | 0.9794 | 0.7062 | 0.7062 | 0.7097 | 0.7100 |
| 25 | 0.9898 | 0.9898 | 0.9898 | 0.9898 | 3.3534 | 3.3532 | 3.3553 | 3.3500 |
| 26 | 0.9920 | 0.9920 | 0.9920 | 0.9920 | 3.4516 | 3.4514 | 3.4531 | 3.4500 |
| 27 | 1.0008 | 1.0008 | 1.0008 | 1.0008 | 4.0788 | 4.0785 | 4.0788 | 4.0800 |
| 28 | 0.9935 | 0.9935 | 0.9935 | 0.9935 | 4.8359 | 4.8356 | 4.8354 | 4.8300 |
| 29 | 0.9903 | 0.9903 | 0.9903 | 0.9903 | 5.1289 | 5.1287 | 5.1289 | 5.1300 |
| 30 | 0.9870 | 0.9870 | 0.9870 | 0.9870 | 6.1850 | 6.1848 | 6.1848 | 6.1800 |
| 31 | 0.9862 | 0.9862 | 0.9862 | 0.9862 | 6.1849 | 6.1848 | 6.1848 | 6.1800 |
| 32 | 0.9859 | 0.9859 | 0.9859 | 0.9859 | 6.1847 | 6.1848 | 6.1849 | 6.1800 |

Abbreviations: BFS, Backward/Forward Sweep; DHPF, Decoupled Harmonic Power Flow; THD ${ }_{U}$, Total harmonic distortion of voltage.

In order to investigate the impact of nonlinear loads and DG units on the power quality of the distribution system, four cases have been considered. In case 1 , which is taken from Heydari et al, ${ }^{32}$ the system contains two nonlinear loads at buses 5 and 26. These loads are the three-phase six pulse converters with active and reactive powers of 1 MW and 0.75 MVAr, respectively. In case 2 , which is taken as the base case, the system contains three nonlinear loads at buses 6,16 , and 30 . In this case, it is assumed that the loads at buses 6,16 , and 30 are the nonlinear PWM adjustable-speed drives (ASD), while the rest ones are linear. In case 3, the system contains three nonlinear loads as in case 2 and one linear DG at bus 27 . Case 4 is the same as case 3, but instead of the linear DG, the nonlinear DG is taken into consideration. As mentioned in Section 2.1, in all cases, the substation voltage magnitude was set to 1 p.u., and it was assumed that the

| Bus | $V_{\text {RMS }}$, p.u. |  |  |  | $\mathrm{THD}_{U}, \%$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BFS | DHPF | ETAP | PCFLO | BFS | DHPF | ETAP | PCFLO |
| 0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.0000 | 0.0001 | 0.0000 | 0.0000 |
| 1 | 0.9986 | 0.9986 | 0.9986 | 0.9986 | 0.2332 | 0.2333 | 0.2412 | 0.2300 |
| 2 | 0.9929 | 0.9929 | 0.9929 | 0.9929 | 1.4882 | 1.4882 | 1.4967 | 1.4900 |
| 3 | 0.9918 | 0.9918 | 0.9918 | 0.9918 | 2.4288 | 2.4289 | 2.4380 | 2.4300 |
| 4 | 0.9911 | 0.9911 | 0.9911 | 0.9911 | 3.4108 | 3.4109 | 3.4208 | 3.4100 |
| 5 | 0.9903 | 0.9903 | 0.9903 | 0.9902 | 6.9336 | 6.9338 | 6.9463 | 6.9300 |
| 6 | 0.9873 | 0.9873 | 0.9873 | 0.9873 | 7.5434 | 7.5435 | 7.5572 | 7.5400 |
| 7 | 0.9827 | 0.9827 | 0.9827 | 0.9827 | 7.5764 | 7.5766 | 7.5902 | 7.5700 |
| 8 | 0.9767 | 0.9767 | 0.9767 | 0.9767 | 7.6478 | 7.6480 | 7.6614 | 7.6400 |
| 9 | 0.9712 | 0.9712 | 0.9712 | 0.9712 | 7.7366 | 7.7369 | 7.7498 | 7.7300 |
| 10 | 0.9704 | 0.9704 | 0.9704 | 0.9704 | 7.7491 | 7.7494 | 7.7624 | 7.7500 |
| 11 | 0.9690 | 0.9690 | 0.9690 | 0.9689 | 7.7733 | 7.7737 | 7.7866 | 7.7700 |
| 12 | 0.9632 | 0.9632 | 0.9632 | 0.9632 | 7.9389 | 7.9393 | 7.9518 | 7.9400 |
| 13 | 0.9611 | 0.9611 | 0.9611 | 0.9611 | 8.0416 | 8.0421 | 8.0542 | 8.0400 |
| 14 | 0.9598 | 0.9598 | 0.9598 | 0.9598 | 8.1311 | 8.1317 | 8.1434 | 8.1300 |
| 15 | 0.9586 | 0.9586 | 0.9586 | 0.9586 | 8.2347 | 8.2353 | 8.2467 | 8.2300 |
| 16 | 0.9569 | 0.9569 | 0.9569 | 0.9569 | 8.5584 | 8.5590 | 8.5693 | 8.5600 |
| 17 | 0.9563 | 0.9563 | 0.9563 | 0.9563 | 8.5586 | 8.5594 | 8.5697 | 8.5600 |
| 18 | 0.9981 | 0.9981 | 0.9981 | 0.9981 | 0.2331 | 0.2332 | 0.2406 | 0.2300 |
| 19 | 0.9945 | 0.9945 | 0.9945 | 0.9945 | 0.2329 | 0.2330 | 0.2399 | 0.2300 |
| 20 | 0.9938 | 0.9938 | 0.9938 | 0.9938 | 0.2329 | 0.2330 | 0.2409 | 0.2300 |
| 21 | 0.9932 | 0.9932 | 0.9932 | 0.9931 | 0.2329 | 0.2330 | 0.2411 | 0.2300 |
| 22 | 0.9894 | 0.9894 | 0.9894 | 0.9894 | 1.4850 | 1.4851 | 1.4935 | 1.4900 |
| 23 | 0.9828 | 0.9828 | 0.9828 | 0.9828 | 1.4815 | 1.4816 | 1.4900 | 1.4800 |
| 24 | 0.9795 | 0.9795 | 0.9795 | 0.9795 | 1.4810 | 1.4811 | 1.4895 | 1.4800 |
| 25 | 0.9920 | 0.9920 | 0.9920 | 0.9920 | 7.3587 | 7.3588 | 7.3714 | 7.3600 |
| 26 | 0.9945 | 0.9945 | 0.9945 | 0.9945 | 7.9562 | 7.9564 | 7.9692 | 7.9500 |
| 27 | 1.0070 | 1.0070 | 1.0070 | 1.0070 | 11.8266 | 11.8269 | 11.8410 | 11.8300 |
| 28 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 12.1933 | 12.1936 | 12.2091 | 12.2000 |
| 29 | 0.9965 | 0.9965 | 0.9966 | 0.9965 | 12.3528 | 12.3532 | 12.3692 | 12.3500 |
| 30 | 0.9934 | 0.9934 | 0.9934 | 0.9934 | 12.9561 | 12.9564 | 12.9747 | 12.9600 |
| 31 | 0.9925 | 0.9925 | 0.9925 | 0.9925 | 12.9555 | 12.9560 | 12.9743 | 12.9600 |
| 32 | 0.9923 | 0.9923 | 0.9923 | 0.9923 | 12.9555 | 12.9560 | 12.9744 | 12.9600 |

TABLE 5 RMS voltages and THD $_{U}$ values of the IEEE 33-bus test system for case 4

Abbreviations: BFS, Backward/Forward Sweep; DHPF, Decoupled Harmonic Power Flow; THD ${ }_{U}$, Total harmonic distortion of voltage.
substation voltage does not contain any harmonic components. The active power and voltage of DG are 2 MW and 1 p.u., respectively. The reactive power output of DG is in the range of -1.5 to 1.5 MVAr. The linear DG produces no harmonics and presented by an inductive reactance. For this study, it is assumed that the linear generator has the subtransient reactance of $20 \%$. The nonlinear converter-based DG is connected to the grid through a six-pulse converter. The typical harmonic spectrums of the nonlinear loads and converter-based DG are taken from Ulinuha and Masoum ${ }^{8}$ and presented in Table 1. All calculations are carried out using the tolerance of 0.00001 p.u. and the maximum number of iterations of 200 .

TABLE 6 Simulation results generated by the proposed BFS method

| Case | Min $V_{\text {RMS }}$, p.u. | Max $V_{\text {RMS }}$, p.u. | Max THD $_{U}$, \% | Fund freq losses, kVA | Harmonic losses, kVA | Total losses, kVA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case 1 | 0.8686 | 1.0000 | 8.1966 | 569.1594 | 8.3614 | 577.5209 |
|  |  |  |  | $+j 365.2993$ | $+j 46.0642$ | $+j 411.3635$ |
| Case 2 | 0.9152 | 1.0000 | 7.9185 | 202.6744 | 5.8961 | 208.5705 |
|  |  |  |  | $+j 135.1391$ | $+j 32.3408$ | $+j 167.4799$ |
| Case 3 | 0.9545 | 1.0008 | 6.1850 | 68.8130 | 4.2855 | 73.0985 |
|  |  |  |  | $+j 52.0144$ | $+j 24.2342$ | $+j 76.2486$ |
| Case 4 | 0.9563 | 1.0070 | 12.9561 | 68.8130 | 17.2709 | 86.0839 |
|  |  |  |  | +j52.0144 | +j107.3251 | +j159.3395 |

Abbreviations: BFS, Backward/Forward Sweep; THD $_{U}$, Total harmonic distortion of voltage.

TABLE 7 The average execution time for different test systems

|  | Execution Time, s |  |
| :--- | :--- | :--- |
| Test System | DHPF | BFS |
| IEEE 33-bus | 0.0239 | 0.0172 |
| IEEE 69-bus | 0.1852 | 0.1283 |
| IEEE 85-bus | 0.2312 | 0.1518 |

Abbreviation: BFS, Backward/Forward Sweep; DHPF, Decoupled Harmonic Power Flow.

## 3.1 | Solution accuracy test

To establish the accuracy of the BFS method, the power flow calculations were also performed using the DHPF method, ${ }^{10}$ Load Flow Analysis module of the ETAP program, ${ }^{23}$ and Full Harmonic Solution module of the PCFLO program. ${ }^{24}$ Simulation results (RMS voltages and $\mathrm{THD}_{U}$ values) are shown in Tables 2-5. Table 2 also contains reference results from Heydari et al ${ }^{32}$ for case 1.

Tables 2-5 indicate that the results obtained by the BFS method are almost identical to those generated by the DHPF method, ETAP, and PCFLO programs. In addition, from the results in Table 2, it appears that the obtained results are in good agreement with reference results. The maximum deviations of the obtained RMS bus voltages and $\mathrm{THD}_{U}$ values from the reference results are $-0.05 \%$ and $-7.59 \%$, respectively.

Summarized results generated by the proposed BFS method are presented in Table 6. In case 1, the total active losses of the system and the maximum $\mathrm{THD}_{U}$ level are 577.5209 kW and $8.1966 \%$, respectively. The reference results reported in Heydari et al ${ }^{32}$ are 578 kW and $8.55 \%$. It is observed that the obtained results are in good agreement with reference results. In the base case, that is, case 2, the total (apparent) power losses of the system are ( $208.5705+j 167.4799$ ) kVA of which ( $5.8961+j 32.3408$ ) kVA presents the losses caused by the harmonics resulting from nonlinear loads. The minimum RMS voltage is 0.9152 p.u., and the maximum $\mathrm{THD}_{U}$ level is $7.9185 \%$. After adding the linear DG unit at bus 27 (case 3), the power losses decrease to $(73.0985+j 76.2486) \mathrm{kVA}$. In addition to reducing of power losses, the installation of the linear DG leads to an improvement of the voltage profile and power quality of the system. The maximum $\mathrm{THD}_{U}$ level in case 3 is reduced from $7.9185 \%$ to $6.185 \%$, and the minimum RMS voltage is increased from 0.9152 to 0.9545 p.u. In the case when DG is connected to the grid through a six-pulse converter (case 4), the total power losses of the system and the maximum $\mathrm{THD}_{U}$ level are $(86.0839+j 159.3395) \mathrm{kVA}$ and $12.9561 \%$, respectively, which shows a decrease of $32.29 \%$ in power losses and an increase of $63.62 \%$ in distortion level of the voltage in comparison with the base case. This increase in the voltage distortion level is due to the additional harmonic currents generated by the DG unit.

## 3.2 | Execution time test

The performance of the proposed method is compared with the DHPF method. ${ }^{10}$ In addition to the IEEE 33-bus test system, two other distribution systems with nonlinear loads, IEEE 69 -bus ${ }^{33}$ and IEEE 85 -bus, ${ }^{34}$ are tested to compare the execution time of these methods. It is assumed that the loads of these test systems comprise a linear part of $80 \%$ and a nonlinear part of $20 \%$ in every bus. All nonlinear loads have the harmonic spectrum of the PWM ASD from Table 1. Table 7 shows a comparison of the average execution time for both methods.

From Table 7, it is evident that the BFS method is faster than the DHPF method. The proposed BFS method does not require the formation and inversion of the bus admittance matrix for each harmonic of interest, and because of that, it is computationally efficient than the DHPF method. In addition, the proposed BFS HPF method is very robust. The convergence has been reached for all tested systems. Depending on the order of the harmonic, the proposed algorithm converges to the final solution after 10 to 120 iteration for each harmonic of interest.

## 4 | CONCLUSIONS

In this paper, a BFS method has been proposed and successfully applied in solving the HPF problem in radial distribution systems with nonlinear loads and DG units. The proposed method has been tested and investigated on three standard IEEE distribution systems, IEEE 33-bus, IEEE 69-bus, and IEEE 85-bus. The main conclusions arising from the results and discussion are as follows:

- The proposed BFS method allows quickly, easily, and accurate calculation of the voltages, currents, power flows and losses, and the harmonic distortions.
- The accuracy and efficiency of the BFS method are successfully verified using the DHPF method, ETAP program, and PCFLO program and well illustrated by its applications to the standard IEEE test systems.
- The maximum absolute deviations of the RMS bus voltages and $\mathrm{THD}_{U}$ values obtained by the BFS method for case 1 from the corresponding reference values ${ }^{32}$ are less than $0.1 \%$ and $8 \%$, respectively.
- It is found that the BFS method can obtain the same solutions as those obtained by the commonly used DHPF method with less computational time.
- The BFS method requires less memory storage than the coupled Newton Raphson and DHPF methods, since the bus admittance matrix inverse is not necessary in the solution procedure.
- The proposed BFS method can be quickly and easily applied to any other distribution system with different types of nonlinear loads and DG units.


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[^0]:    Some of the authors of this publication are also working on these related projects:

[^1]:    Abbreviations: BFS, Backward/Forward Sweep; DHPF, Decoupled Harmonic Power Flow; THD $_{U}$, Total harmonic distortion of voltage.

