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AVERAGE BIT ERROR RATE CALCULATION BASED ON USING PIECEWISE SPLINE APPROXIMATION

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In this paper, an approximation of the average bit error rate (ABER) probability expression, standard performance measure of wireless transmission over fading channels using linear spline functions with corresponding number of segments has been provided. Further, possible approximation improvement obtained by appliance of composite piecewise spline (combination of linear and quadratic) functions has also been analyzed. For the generality purpose multipath fading that occurs through the propagation was modeled with alpha-kappa-mu distribution model, which includes as special cases other fading models, so this analysis obtains a high level of generality. It has been shown that this approximation provides a high level of accuracy in the wide range of observed conditions so this model can be applied for high-accuracy prediction of the performance measures for various wireless services.

1. INTRODUCTION

One of the most important wireless transmission performance criterions that is related to various quality-of-service (QoS) issues is average bit error rate (ABER) at the reception [1]. However, the analytical derivation of closed-form expressions is often very difficult task, and in some non-coherent modulation cases even impossible. Namely, ABER at the reception for noncoherent binary signaling is derived by averaging the conditional error probability over the probability density function (PDF) of the wireless signal random envelope at the reception, by taking into account parameters that are related to the modulation format that has been used for transmission. Most recently such problem has been observed in [2] and [3]. In [2] BER performances at reception over alpha-kappa-mu fading channels were analyzed, but only when spatial modulation *M*-quadrature amplitude modulation (M-QAM) format has been applied binary differential phase shift keying (BDPSK) modulation format performances has not been considered most probably due to difficulties that occur in the analytical derivation of closed-form expressions for such case. However even for M-QAM format case closed-form expressions are obtained using both Craigs approximation and Gauss-Legendre quadrature approximation. However in such case approximation error occurs which highly depends on the number of weights, roots and nodes of Gauss-Legendre quadrature polynomial. In [3] both Gauss-Hermite integration and Q-function approximation are used in order to obtain closed-form expressions for BER over alpha-kappa-mu fading channels. Here two approximation errors accumulate: error from Gauss-Hermite integration and error from Q-function approximation.

Major obstacle in wireless communication is occurrence of multipath fading, caused by reflections of observed signal from various objects, in the way that at the reception those randomly delayed, reflected and scattered signal components combine in constructive, or more often in destructive manner [4]. Various fading models have been used to model the random process of faded signal amplitude variation. Recently general, α-κ-μ fading model has been presented, expressed in the function of three fading parameters, namely parameter α related to the nonlinearity of the propagation environment; parameter μ, denoting the number of multipath clusters in the environment through which signal propagates, and parameter κ defining to the ratio between the dominant line-of-sight (LOS) component and scattered components [5]. As a general model, it also reduces to other previously well-known fading models, as in special cases.

In order to reduce complex computations a need for accurate approximation and ABER values prediction arises. Efficient approach for such ABER approximation that has been already exploited in literature is interval approximation of Q-function [6] or composite Q-function approximation [7]. For the evaluation of the ABER over fading channels, expression for conditional BER (conditioned over fading statistics which impairs the communication) should be averaged over the probability density function (PDF) of the fading channel amplitude. In many such cases the averaging integral includes either the Gaussian Q-function, either directly related functions: error function, erf(x), and/or complementary error function erfc(x). However, in this paper we will present novel approach for accurate ABER approximation based on spline approximation of fading channel amplitude PDF statistics.

The major advantage of spline approximation is its local property, *i.e.* property that changing observed data in some small area affects the approximation in the same area. Spline function in that way consists of polynomial pieces on subintervals joined with certain continuity conditions, and is in particular piecewise linear interpolation. Since ABER expression depends on PDF of the received random envelope, we will perform the approximation of the received signal PDF using the linear spline functions with the support region divided into $L = 4$ and $L = 8$ segments, respectively. It will be shown that reception performances obtained by presented approximation closely follow reception performance obtained by complex numerical evaluation. Further, possible approximation improvement

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obtained by the appliance of composite piecewise spline (combination of linear and quadratic) functions has also been analyzed. Based on the obtained spline approximation of ABER, wireless link designing process could be performed for observed non-homogenous, non-linear general fading environment. Obtained approximation properties suggest the possibility of exploiting the presented approximation in various wireless communication applications.

2. SYSTEM MODEL

Various fading models assume a resultant homogeneous diffuse scattering field, from randomly distributed scatters. However, surfaces are often spatially correlated and they characterize non-linear environment. Exploring the fact that resulting envelope could be a nonlinear function of the sum of multipath components, novel general α-κ-μ distribution for short-time fading model was recently presented. PDF expression for the random envelope process is given in the form of [5]:

$$f_R(R)=\frac{\alpha k^{\frac{1-\mu}{2}}(1+k)^{\frac{1+\mu}{2}}\mu R^{\frac{\alpha(1+\mu)}{2}-1}}{\exp(\mu k)\Omega^{\frac{1+\mu}{2}}}\times \times\exp\left(-\frac{\mu(1+k)R^{\alpha}}{\Omega}\right)I_{\mu-1}\left(2\mu\sqrt{k(1+k)\frac{R^{\alpha}}{\Omega}}\right) \quad (1)$$

with $\Omega=E[R^{\alpha}]$, denoting the desired signal average power, $I_n(x)$ denoting the n-th order modified Bessel function of the first kind, Eq. (8.406) from [8]. Parameters α, k and μ are detailly explained in [5].

As a general model, it also reduces to other models, as it special cases. The κ-μ fading model can be obtained from the α-κ-μ model by setting α = 2. Further, Nakagami-*m* fading model can be obtained by setting κ = 0. Similarly, Ricean fading model can be obtained by setting μ = 1 in obtained κ-μ model. The α−μ fading model can also be obtained from the α-κ-μ model by setting κ = 0. Other fading models like Weibull and Rayleigh are just singularities of above mentioned fading models [9].

Possibility of using spline approximations for PDFs approximations have been observed in literature recently, [10–12]. In [10] an N-order spline approximation to a some PDFs based on N statistical moments of the distribution has been presented. However this approximation is based on distribution properties, so method convergence is questionable, and is more complex and of higher order comparing to spline approximation presented in this work. In [11] Weibull PDF has been approximated based on the piecewise cubic polynomial spline. In our work here we are observing more complex PDF than Weibull with quadratic spline approximation.

The spline function is a function that consists of polynomial pieces joined together with certain smoothness conditions. A simple example is the polygonal function (or spline of degree 1), whose pieces are linear polynomials joined together to achieve continuity [13]. In the theory of splines, the points $x_0, x_1,..., x_L$ at which the function changes its character are termed knots [13]. Such a function appears somewhat complicated when defined in explicit terms. Accordingly, we consider the following definition of a linear polynomial $S(x)$ [13]:

$$S(x)=\begin{cases}S_0(x), & x\in[x_0,x_1],\\ S_1(x), & x\in[x_1,x_2],\\ \vdots & \\ S_{L-1}(x), & x\in[x_{L-1},x_L],\end{cases} \quad (2)$$

where

$$S_i(x)=a_i(x)+b_i\,. \quad (3)$$

Obviously, $S(x)$ is a piecewise linear function. For the given knots $x_0, x_1,..., x_L$ and coefficients $a_0, b_0, a_1, b_1,\ldots, a_{L-1}, b_{L-1}$, the evaluation of $S(x)$ at a specific x performs by first determining the interval that contains x and then by using the appropriate linear function for that interval. If the function S defined by (2) is continuous, we call it a first-degree spline. It is characterized by the following three properties [13].

Definition 1. A function S is called a spline of the first-degree if:

1. The domain of S is an interval $[a, b]$;
2. S is continuous on $[a, b]$;
3. There is a partitioning of the interval $a=x_0<x_1<...<x_L=b$ such that S is a linear polynomial on each subinterval $[x_i, x_{i+1}]$.

Outside the interval $[a, b]$, $S(x)$ is usually defined to be the same function on the left of a as it is on the leftmost subinterval $[x_0, x_1]$ and the same on the right of b as it is on the rightmost subinterval $[x_{L-1}, x_L]$ [13]. In other words, $S(x)=S_0(x)$ when $x<a$ and $S(x) = S_{L-1}(x)$ when $x > b$. The first-degree spline, also called the polygonal function, is consisted of line segments that are connected so that given function is continuous. As already mentioned, the points where the function changes its shape are called knots [13].

The approximate function $p(r)$, by which a PDF from (1) is approximated in this paper, for the number of segments L, has the following form [13]:

$$p(r)=\begin{cases}f(r_1)+m_1(r-r_1), & r\in[0,r_1],\\ f(r_i)+m_i(r-r_i), & r\in[r_{i-1},r_i], \quad i=2,...,L,\end{cases} \quad (4)$$

where m_i is the coefficient of direction of the line given by:

$$m_i=\frac{f(r_i)-f(r_{i-1})}{r_i-r_{i-1}}, \quad (5)$$

where $i = 1,..., L$.

Definition 2. A function Q is called a spline of the second-degree if:

1. The domain of Q is an interval $[a,b]$;
2. Q and Q' are continuous on $[a,b]$;
3. There are points t_i (called knots) such that $a=t_0<t_1<...<t_n=b$ and Q is a polynomial of degree at most two on each subinterval $[t_i, t_{i+1}]$ [14].

Accordingly, we consider the following definition of a quadratic polynomial $Q(x)$ [14, 15]:

$$Q(x)=\begin{cases}Q_0(x), & x\in[x_0,x_1],\\ Q_1(x), & x\in[x_1,x_2],\\ \vdots & \\ Q_{L-1}(x), & x\in[x_{L-1},x_L]\end{cases}, \quad (6)$$

where

$$Q_i(x)=c_{0i}x^2+c_{1i}x+c_{2i}\,. \quad (7)$$

ABER in observed environment, when transmission is carried out over corresponding modulation format can be determined according to [16]:

$$P_e = \int_0^\infty P_{e|y}(e|y)p(y)\mathrm{d}y, \quad (8)$$

with conditional ABER, $P_{e|y}(e|y)$, depending on modulation form being used. Namely, for non-coherent modulation formats, *i.e.* for noncoherent frequency shift keying (NCFSK) and binary differential phase shift keying (BDPSK) $P_{e|y}(e|y)=1/2\exp(-gy)$, with $g=1/2$ and $g=1$, respectively, while for coherent modulation formats, *i.e.* coherent frequency shift keying (CFSK) and coherent phase shift keying (CPSK) $P_{e|y}(e|y)=1/2\,\mathrm{erfc}\left(\sqrt{g\cdot y}\right)$, with $\mathrm{erfc}(x)$ denoting complementary error function, and $g = 1/2$ and $g = 1$, respectively.

3. NUMERICAL RESULTS

In this section results obtained for $L = 4$ and $L = 8$ spline approximations of α-κ-μ fading envelope PDF expression are presented.

Spline intervals and coefficients for $L = 4$ approximation calculated according to equation (9) for considered set of fading model parameters are presented in Table 1.

$$\left.\frac{\partial p(r)}{\partial r}\right|_{r=R_{\max}} = 0; \quad R_{i+1}-R_i=\Delta, \quad i\in(0,L-1);$$
$$\Delta = \frac{R_{\max}-R_{\min}}{L}; \quad L=4; \quad (9)$$
$$f_1(r)=\begin{cases} S_1(r)=a_1R+b_1, & r\in[R_{\min},R_1] \\ S_2(r)=a_2R+b_2, & r\in[R_1,R_{\max}] \\ S_3(r)=a_3R+b_3, & r\in[R_{\max},R_3] \\ S_4(r)=a_4R+b_4, & r\in[R_3,R_4] \end{cases}$$

Table 1

Spline intervals and coefficients for L=4 approximation

α=1, κ=1,μ=2, Ω=1	a_i	b_i
(R_{min}=0, R_1=0.665)	0.43944	0
(R_1=0.665,R_2=R_{max}=1.33)	0.12085	0.21186
(R_2=R_{max}=1.33,R_3=2.3275)	–0.10778	0.51595
(R_3=2.3275, R_4=5.367)	–0.08098	0.45357

In Fig. 1, graphical presentation of $L = 4$ spline approximation from Table 1 is given. The very good match between original and approximated characteristics can be seen.

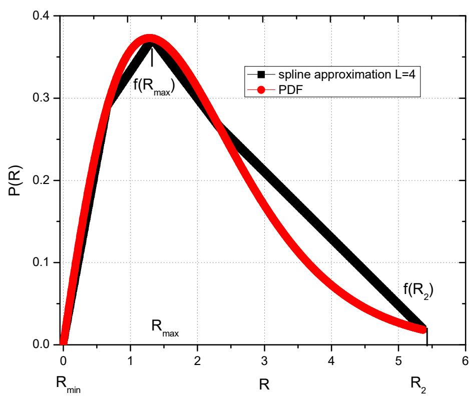

Fig. 1 – Graphical presentation of $L = 4$ linear spline approximation.

Now, we will observe how mentioned ABER wireless performance criterion, can be estimated based on performed $L = 4$ spline approximation of fading envelope PDF expression, for the case when BDPSK modulation is performed. For the same set of observed fading parameters relative error in ABER evaluating is calculated according to:

$$P_{e_1} = \int_0^\infty p(r)\frac{1}{2}\exp(-gr)\mathrm{d}r;$$
$$P_{e_2} = \int_0^\infty f_1(r)\frac{1}{2}\exp(-gr)\mathrm{d}r, \quad (10)$$
$$\left|\frac{P_{e_1}-P_{e_2}}{P_{e_1}}\right| = 0.0444.$$

This result represents significant performance improvement comparing to the case of $L = 2$ linear spline approximation, with relative ABER error of 0.1973. Another improvement could be obtained by increasing the number of segments for linear spline approximation on $L = 8$. For such case ABER relative error is reduced from 0.0444 to 0.03867. The conclusion arises that a further increase in the number of segments for linear spline approximation would lead to further relative error reduction and performance improvement, similarly like in [17] where linear spline was observed and in [18] where SQNR improvement is reached with increase in number of segments. However, besides memory limitations, there are other constraints that are related to number of segments increase. So, for a limited number of segments, there is a need to optimize the selection of approximation spline function. One of the approaches that can be used is combining spline functions of various order into one composite piecewise spline function. Let us apply following piecewise spline algorithm: First we will divide region into $L = 8$ segments, ready for linear spline approximation, but then we will apply linear spline approximation in the first segment, and in the last 3 segments of the region. In total 4 segments are approximated by a linear spline. Then, from 4 non-approximated segments 2 neighbor segments are merged 1 novel segment is obtained. For this segment, quadratic spline approximation has been carried out, according to Eqs. (6) and (7).

For considered set of fading model parameters this composite piecewise spline approximation can be performed according to equation (11):

$$\left.\frac{\partial p(r)}{\partial r}\right|_{r=R_{\max}} = 0; \quad R_{i+1}-R_i=\Delta, \quad i\in(0,L-1);$$
$$\Delta = \frac{R_{\max}-R_{\min}}{L}; \quad L=5; \quad (11)$$
$$f_2(r)=\begin{cases} S_1(r)=a_1R+b_1, & r\in[R_{\min},R_1] \\ Q_1(r)=c_0r^2+c_1r+c_2, & r\in[R_1,R_{\max},R_2] \\ S_2(r)=a_3R+b_3, & r\in[R_2,R_3] \\ S_3(r)=a_4R+b_4, & r\in[R_3,R_4] \\ S_4(r)=a_5R+b_5, & r\in[R_4,R_5] \end{cases}$$

Spline intervals and coefficients for considered set of fading model parameters are presented in Table 2, while the graphical presentation of composite piecewise spline approximation is given in Fig. 2.

Table 2

Spline intervals and coefficients for composite piecewise spline approximation

α=1, κ=1, μ=2, Ω=1	a_i		b_i
(R_{min}=0, R_1=0.665)	0.43944		0
(R_1=0.665, R_{max}=1.33, R_2=2.3275)	c_0	c_1	c_2
	–0.1375	0.3952	0.0902
(R_2=2.3275, R_3=3.34089)	–0.13368		0.57622
(R_3=3.34089, R_4=4.35429)	–0.07680		0.38621
(R_4=4.35429, R_5=5.36768)	–0.03319		0.19629

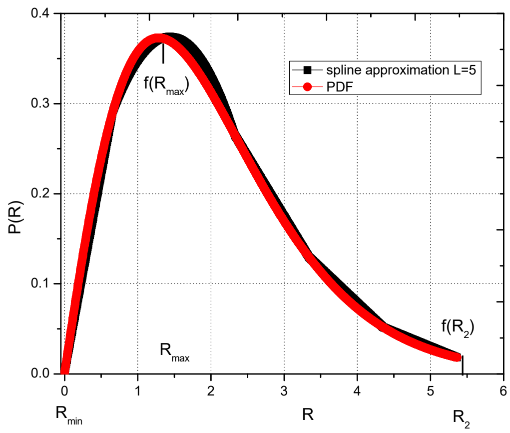

Fig. 2 – Graphical presentation of composite piecewise spline approximation from Table 2.

From Fig. 2 one can see even better match between original and approximated characteristics than presented at Fig.1. After calculating relative error according to:

$$P_{e_1} = \int_0^\infty p(r)\frac{1}{2}\exp(-gr)\mathrm{d}r\,;$$

$$P_{e_2} = \int_0^\infty f_2(r)\frac{1}{2}\exp(-gr)\mathrm{d}r\,, \qquad (12)$$

$$\left|\frac{P_{e_1} - P_{e_2}}{P_{e_1}}\right| = 0.029248$$

and comparing it with relative error obtained for linear spline approximation for L = 8 segments of 0.03876, it is clear that significant improvement has been achieved while using reduced number of segments.

4. CONCLUSIONS

It has been shown in this paper that very good prediction of some wireless transmission performance measures behavior can be accomplished, based on using spline approximation of observed fading envelope PDF model expression for each given set of observed model parameters. The used type of spline approximation depends of the demanded accuracy of parameter estimation. The possibility of exploiting the presented method in various communication applications arises for estimating system performances that can be determined based on statistical models of transmitted signals.

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